Power Guarantee for Electric Systems Using Real-Time Scheduling

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Abstract—Modern electric systems, such as electric vehicles, mobile robots, nano satellites, and drones, require to support various power-demand operations for user applications and system maintenance. This, in turn, calls for advanced power management that jointly considers power demand by the operations and power supply from various sources, such as batteries, solar panels, and supercapacitors. In this article, we develop a power scheduling framework for a reliable energy storage system with multiple power-supply sources and multiple power-demand operations. Specifically, we develop offline power-supply guarantee analysis and online power management. The former provides an offline power-supply guarantee such that every power-demand operation completes its execution in time while the sum of power required by individual operations does not exceed the total power supplied by the entire energy storage system at any time; to this end, we develop a plain power-supply analysis as well as its improved version using real-time scheduling techniques. On the other hand, the latter efficiently utilizes the surplus power available at runtime for improving system performance; we propose two approaches, depending on whether future scheduling information of power-demanding tasks is available or not. For evaluation, we perform simulations to evaluate both the plain and improved analyses for offline power guarantee under various synthetic power-demand operations. In addition, we have built a simulation model and demonstrated that the proposed framework with the offline analysis and online management not only guarantees the required power-supply, but also enhances system performance by up to 56.49 percent.

Index Terms—Offline power-supply guarantee, online power management, real-time scheduling, electric systems

1 INTRODUCTION

Recent advances on cyber-physical systems (CPS) require various mechanical/electrical operations for many physical platforms, such as electric vehicles, mobile robots, nano satellites, and drones. For example, a drone needs to (i) operate the motors to fly, (ii) power sensors, coolers, and communication modules, (iii) operate camera(s) and stepper motors to take pictures and deliver parcels, and (iv) perform the computation necessary to maintain its stability (see Fig. 13 in the supplement, which can be found on the Computer Society Digital Library at http://doi.ieeecomputersociety.org/10.1109/TPDS.2020.2977041). These multiple power-demand operations impose different power demands on the system and may be triggered at different times, requiring the system to effectively provide time-varying supply of power [1], [2], [3], [4], [5], [6]. Existing studies on the power scheduling problem focused on the reduction of peak power by scheduling multiple power-demand operations and analysis thereof [7], [8], [9], [10].

However, a complete solution to the power scheduling problem also needs to characterize energy storages/sources because it affects power capability for the power-demand operations. We address this need by targeting hybrid energy storage systems (HESs) comprised of multiple power-supply sources and storages, such as batteries, supercapacitors, and renewable energy sources, whose concept and implementation have been proposed and explored in [4], [11], [12], [13], [14], [15], [16]. A combination of high-energy-density batteries and high-power-density supercapacitors enables the system to supply the required time-varying power for a longer time, improving system sustainability. Moreover, renewable power sources (e.g., solar, wind and geothermal energy) enhance the system’s power capacity and reduce the load intensity on batteries and supercapacitors, thus prolonging their lifetime [17], [18], [19], [20], [21].

Researchers have studied the optimal design of a HESS to meet the power demands at the minimum cost [12], [22], [23], [24], [25]. They first explored possible storage configurations (i.e., size, type, connection) of multiple power-source supplies and storages, and then selected the best configuration by comparing the performance and the energy production cost. Search for an optimal design iterates to generate candidate configurations until one of the candidates satisfies the predefined performance and cost criteria [23]. However, these approaches do not guarantee power-sufficiency during operation, as the electric load and criteria are determined based on the load history or the designer’s experience and intuition.
To ensure the power sufficiency, we develop a power scheduling and analysis framework for a reliable energy storage system with multiple power-supply sources and multiple power-demand operations, achieving the following goals.

G1. **Offline power-supply guarantee**: We provide an offline guarantee to complete every operation before its deadline (i.e., operation-level power guarantee) while keeping the amount of power supplied to the entire system no less than the sum of power required by individual operations at any time instant (i.e., system-level power guarantee).

G2. **Online power management**: We develop online power management that effectively utilizes the difference between the worst-case supplied/demanded power and the actual one for improving system performance.

This is a typical CPS problem in that we should address power scheduling and its analysis in the cyber space, based on comprehensive understanding of the physical characteristics of power supply and demand.

To achieve G1, we find similarities between the power-supply guarantee problem and a real-time scheduling problem that determines the execution order of real-time tasks; while the operation-level power guarantee corresponds to the task-level deadline satisfaction, the system-level power guarantee matches the computing platform’s capacity constraint. Using the techniques of real-time scheduling, we solve the power-supply guarantee problem in two steps. First, we address the case of multiple power-demand operations with a single, uniform power-supply source (e.g., a battery pack), and develop a scheduling framework and offline power guarantee analysis. For the analysis, we develop a plain analysis that only considers each operation’s executable interval (from its release to deadline). We then improve the plain analysis by iteratively claiming each operation’s slack (the gap between the finishing time and the deadline). Second, based on the scheduling and analysis framework, we address the general problem—multiple power-demand operations with multiple power-supply sources. To this end, we develop two scheduling frameworks to utilize additional sporadic power-supply sources: one for sharing additional power-supply sources by all power-demand operations (called the uniform supply approach), and the other for assigning the sources to only some of power-demand operations (called the dedicated supply approach). Our solution for G1 not only demonstrates that the technique of real-time scheduling helps solve a CPS problem, but also addresses the design problem of a HESS by finding a combination of energy storages at minimum cost.

For the evaluation of G1, we synthesize various power-demand operations and compare the power-guarantee performance of the combination of (i) analysis version (i.e., the plain and improved analyses), and (ii) the existence of additional power-supply sources and the framework type for handling them (i.e., no additional supply, uniform and dedicated supply). We also consider two different sets of additional power-supply sources for the uniform and dedicated supply approaches. We have the following observations for the performance of power guarantee. First, the improved analysis dominates the plain one, meaning that the operation set power-guaranteed by the latter is always power-guaranteed by the former. Second, the performance difference between the former and the latter varies with (ii) and the sets of additional power-supply sources. Third, the dedicated supply outperforms the uniform supply for most cases, but not always.

In addition to making the offline power-supply guarantee (G1), we develop online power management (G2), which aims at increasing the utilization of the energy generated by renewable power-supply sources. That is, scheduling multiple power-supply sources and power-demand operations according to G1 necessarily yields surplus energy since G1 considers the worst case, i.e., the largest power demand and the smallest power supply. While we can utilize the surplus energy for improving various performance aspects of a HESS, we focus on reduction of the peak power supplied by the main energy storage (i.e., a battery pack) to improve its capacity and lifetime [15], [24], [26], [27], [28], [29]. We propose two online power management approaches: static-demand-based and predicted-demand-based approaches. The former operates effectively when the future scheduling information of power-demand operations is known, while the latter does for a more general case where only the past scheduling information is available.

We focus on a case study for a HESS-powered system to evaluate the proposed solutions for G1 and G2 together. The system for the case study consists of power-consuming components such as wheel motors, stepper motors, coolers, sensors and converters, whose operations are controlled by an application. The system is powered by a HESS consisting of Lithium-ion batteries, ultra-capacitors (UCs), and solar panels. Using the offline power-supply guarantee analysis, we can determine the minimum battery capacity without compromising power-supply guarantee. Our experimental results using a simulation model show that the system with our online power management schemes not only supplies a sufficient amount of power to the components during their operation, but also reduces the battery’s peak power by 56.49 percent.

In summary, this paper makes the following main contributions:

- Design of a power scheduling and analysis framework that consists of (i) offline power-supply guarantee, which is the first power guarantee analysis applicable to the design of a HESS, and (ii) online power management for a HESS to enhance the energy storage’s performance,
- Demonstration of the effectiveness of the proposed framework via simulations with synthesized power-demand operation sets and a realistic simulation model for a case study, and
- Solution of an important CPS problem using real-time scheduling techniques.

The paper is organized as follows. Section 2 describes the characteristics of power-supply sources and power-demand operations, and then formally states our main problem. Sections 3 and 4 present the power scheduling and analysis framework for the offline power-supply guarantee, while Section 5 details online power management. Section 6 evaluates the offline power-supply guarantee using simulation. Section 7 focuses on a case study and evaluates the power...
sufficiency and peak power reduction achieved by our solutions. Finally, the paper concludes with Section 8.

## 2 System Model and Problem Statement

### 2.1 System Model

As the first step for offline power-supply guarantee and online power management, we need to investigate the characteristics of power-demand operations and power-supply sources. Since our target power-demand operations and power-supply sources are the same as our preliminary conference version [30], we describe them in the supplement, available online, and briefly summarize here.

A system has a set of mechanical/electrical operations \( \lambda = \{ \lambda_1, \lambda_2, \ldots, \lambda_n \} \), where \( n \) is the number of operations.\(^1\) The power usage of \( \lambda_i \) is modeled by the minimum inter-arrival time \( T_{i_d} \), its maximum power consumption \( P_{i_d} \), and the maximum length of execution \( L_{i_d}^s \).

We consider a battery-centric hybrid energy storage system (HESS) that is comprised of three parts: (i) an energy-dense lithium-ion battery pack \( \Gamma_{bat} \); (ii) a set of auxiliary renewable energy sources \( \Gamma = \{ \Gamma_1, \Gamma_2, \ldots, \Gamma_n \} \); and (iii) a power-dense energy buffer \( \Gamma_{uc} \). Each energy source \( \Gamma_i \) is modeled with the maximum inter-arrival time \( T_{i_d}^s \), the minimum supplied power \( P_{i_d}^s \), and the minimum length of supply duration \( L_{i_d}^s \).

Throughout the paper, \( P(t) \) implies the actual power demand/supply at \( t \), and \( P \) represents the maximum power demand or minimum power supply. Also, we use the term of an “operation” for an “instance” of the operation, when no ambiguity arises.

### 2.2 Problem Statement

Using the characteristics of power-demand operations and power-supply sources described in Section 2.1, we formally state the problem to be solved as:

**Given** a set of mechanical/electrical operations \( \lambda \), the battery pack \( \Gamma_{bat} \), the energy buffer \( \Gamma_{uc} \), a set of auxiliary renewable energy sources \( \Gamma \), and the operation interval \([0, t_{max}]\), determine \( P_{bat}^s(t) \) and \( \{ P_{i_d}^s(t) \}_{i=1}^{n} \) for all \( t \in [0, t_{max}] \) to achieve the following objective function.

**Minimize(S0)**

\[
\int_{0}^{t_{max}} P_{bat}(t) + P_{bat-loss}(t) \, dt,
\]

**Subject to**

1. \( \sum_{i=1}^{n} P_{i_d}^s(t) - P_{i_d}^d(t) - \sum_{i=1}^{n} P_{i_d}^s(t) \leq P_{bat}^s(t) \leq P_{uc}^s \)

2. Every instance of each operation \( \lambda_i \in \lambda \) finishes its execution within \( T_{i_d} \) time units after its release.

\( P_{bat}^s(t) \) in S0 denotes the amount of power dissipation at \( t \) caused by the power supply of the battery pack; in other words, we lose \( P_{bat-loss}(t) \) of power due to supplying \( P_{bat}^s(t) \) of power from the battery pack at \( t \). Therefore, S0 implies the amount of energy used and dissipated by the battery pack; since the former (i.e., the amount of energy used) is not a control knob, we should reduce the latter (i.e., the amount of energy dissipated) using a simple circuit-based battery model [31]:

\[
P_{bat-loss}(t) = I_{bat}(t) \times R_{bat}(t),
\]

where \( I_{bat}(t) \) is a battery’s internal resistance, and \( I_{bat}(t) \) is the battery’s discharge/charge current for supplying \( P_{bat}(t) \). Since power dissipation is quadratically proportional to discharge/charge battery current, the minimization of battery peak current may reduce energy dissipation of the battery pack, potentially extending the battery operation-time.

## 3 Scheduling of Multiple Power-Demand Operations with a Uniform Supply

In this section, we present how to schedule a set of power-demand operations (\( \lambda \)) when the battery pack (\( \Gamma_{bat} \)) is the sole power-supply source. We first develop a scheduling framework that considers the characteristics of \( \lambda \) and \( \Gamma_{bat} \) described in Section 2.1. Under this scheduling framework, we then develop an offline power guarantee analysis that determines whether every operation in \( \lambda \) powered by the battery pack is performed before its deadline without suffering any power shortage. Finally, we derive an improved version of the offline power guarantee analysis, which accommodates significantly more (guaranteed) power-demand operations under the same scheduling framework.

### 3.1 Scheduling Framework

We would like to schedule a set of multiple operations (\( \lambda \)) so as to achieve a system-level and operation-level power guarantee under the uniform supply from the battery pack \( \Gamma_{bat} \). By “system-level power guarantee,” we mean that the sum of power demand at any time instant should be no larger than maximum power capability of the battery pack, i.e., \( \sum_{\lambda \in \lambda} P_{\lambda}(t) \leq P_{bat}^s \), which is equivalent to S1.\(^2\) To achieve the operation-level power guarantee, every operation should receive sufficient power within its period, which is S3.

Our scheduling framework employs the work-conserving policy based on the worst-case power demand and operation-level fixed-priority scheduling policy. The former implies that an operation \( \lambda_i \) can start to execute as long as its worst-case power demand \( P_{\lambda_i}^d \) (as opposed to the actual power demand \( P_{\lambda_i}(t) \)) is no larger than the difference between the battery capability \( P_{bat}^s \) and the sum of the worst-case power demand of currently-executing operations \( \sum_{\lambda \in \lambda_{run}} P_{\lambda}^d \). The latter implies that the scheduling framework prioritizes the operations that satisfy the above condition.

Since each operation exhibits the non-preemptive behavior as described in Section 2.1, an operation can start its execution only when it is released or the other operation is finished, and each operation, once started, continues its

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1. Throughout the paper, we use the superscripts \( d \) and \( s \) for power demand and power supply, respectively.
2. Since this section considers the sole supply of \( \Gamma_{bat} \), we remove terms of the energy buffer and a set of renewable power-supply sources in S1.
execution until the completion. Therefore, the scheduling framework can be expressed (as in Algorithm 1) by describing actions for the situations. Lines 1–3 update the ready queue \( Q_{\text{ready}} \) that contains operations ready to execute, and the running queue \( Q_{\text{run}} \) for currently-executing operations. Lines 4–9 select operations to be started and move them from \( Q_{\text{ready}} \) to \( Q_{\text{run}} \). Although the scheduling framework prioritizes operations based on their priorities, a lower-priority operation \( \lambda_j \) in \( Q_{\text{ready}} \) can start its execution earlier than a higher-priority operation \( \lambda_h \) in \( Q_{\text{ready}} \), if the remaining power capability of the system (i.e., \( P_{\text{bat}}^a - \sum_{t \in Q_{\text{run}}} P_t^a \)) is larger than \( P_k^a \) but, no larger than \( P_f^a \), as described in line 5.

### Algorithm 1. Scheduling Framework

The following steps are performed whenever at least one mechanical/electrical operation is finished or released at \( t \):

1. For each \( \lambda_t \) finished at \( t \), \( Q_{\text{run}} \leftarrow Q_{\text{run}} \setminus \{ \lambda_t \} \)
2. For each \( \lambda_t \) released at \( t \), \( Q_{\text{ready}} \leftarrow Q_{\text{ready}} \cup \{ \lambda_t \} \)
3. Sort \( Q_{\text{ready}} \) by given operation-level fixed-priorities
4. for \( \lambda_t \in Q_{\text{ready}} \) (a higher priority operation is chosen earlier)
5. if \( P_k^a \leq P_{\text{bat}}^a - \sum_{t \in Q_{\text{run}}} P_t^a \) then
6. \( Q_{\text{run}} \leftarrow Q_{\text{run}} \cup \{ \lambda_t \} \)
7. \( Q_{\text{ready}} \leftarrow Q_{\text{ready}} \setminus \{ \lambda_t \} \)
8. end if
9. end for

### 3.2 Offline Power Guarantee Analysis

Since each operation is non-preemptive, we need to check if each operation \( \lambda_t \) can start its execution no later than \((T_k^a - L_k^a)\) time units after its release; once it starts to execute, it completes execution within its period without any preemption. Therefore, we focus on the interval of length \((T_k^a - L_k^a + \epsilon_i)\), which begins at the time of \( \lambda_t \)'s release, and check whether the sum of energy consumed by other operations within the interval is strictly less than \((P_{\text{bat}}^a - P_k^a + \epsilon_p) \cdot (T_k^a - L_k^a + \epsilon_i)\), where \( \epsilon_i \) and \( \epsilon_p \) denote the time and power quantum (the smallest unit). Since the smallest power that prevents \( \lambda_t \) from execution at each time instant is \((P_{\text{bat}}^a - P_k^a + \epsilon_p)\), the above condition guarantees the start of \( \lambda_t \)'s execution to be no later than \((T_k^a - L_k^a)\) time units after its release.

The remaining step is then to calculate \( I_{k-i}(\ell) \), the amount of energy demanded by instances of \( \lambda_t \) within an interval of length \( \ell \) that contributes to prevention of \( \lambda_t \) from starting its execution. Note that for calculation of \( I_{k-i}(\ell) \), we limit the power demand at each time instant to \((P_{\text{bat}}^a - P_k^a + \epsilon_p)\), since we need to know whether the sum of all power demands at each time instant is no smaller than \((P_{\text{bat}}^a - P_k^a + \epsilon_p)\). Now, we will describe how to calculate an upper bound of \( I_{k-i}(\ell) \).

First, if \( \lambda_t \) has a higher priority than \( \lambda_h \), then the upper-bound of \( I_{k-i}(\ell) \) is the maximum energy demanded by \( \lambda_t \) in an interval of length \( \ell \), which is calculated as

\[
W_{k-i}(\ell) = \min\left(P_i^a, P_{\text{bat}}^a - P_k^a + \epsilon_p\right) \times \min\left(\ell, N_i(\ell) \cdot L_i^a + \min\left(L_i^a, \ell + T_i^a - L_i^a - N_i(\ell) \cdot T_i^a\right)\right),
\]

where \( N_i(\ell) = \left\lceil \frac{\ell + T_i^a - L_i^a}{T_i^a} \right\rceil \). This calculation is similar to the workloads calculation of real-time scheduling [32]. Briefly, \( N_i(\ell) \) implies the number of instances of \( \lambda_t \), and each of their periods is completely included within the interval of length \( \ell \) (including the first instance of \( \lambda_t \)), which contributes \( \min\left(P_i^a, P_{\text{bat}}^a - P_k^a + \epsilon_p\right) \cdot N_i(\ell) \cdot L_i^a \) of energy to \( W_{k-i}(\ell) \). The second part of \( W_{k-i}(\ell) \) represents the contribution of the operation whose period is partially included in the interval of length \( \ell \).

Second, if \( \lambda_t \) has a lower priority than \( \lambda_h \), then we consider two sub-cases. Since each operation is non-preemptive, \( \lambda_t \) can execute before the start of \( \lambda_h \)'s execution, if \( \lambda_t \) starts its execution before the release of \( \lambda_h \). In this case, the energy demand is upper-bounded by \( \min\left(P_i^a, P_{\text{bat}}^a - P_k^a + \epsilon_p\right) \cdot \min\left(L_i^a, \ell, T_i^a\right) \), which is an upper-bound of \( I_{k-i}(\ell) \) for the first sub-case of \( P_i^a \geq P_k^a \). If \( P_i^a < P_k^a \), then an upper-bound of \( I_{k-i}(\ell) \) can be larger than the first sub-case. That is, due to our worst-case-based work-conserving policy, it is possible for \( \lambda_h \) to start its execution before the start of \( \lambda_t \)'s execution, whenever \( \lambda_h \) does not satisfy Line 5 of Algorithm 1 but \( \lambda_t \) does. In this case, we use the general upper-bound \( W_{k-i}(\ell) \) as an upper-bound of the second sub-case.

Combining all the results discussed so far, we develop a power guarantee analysis as follows.

### Lemma 1

Suppose every \( \lambda_t \in \lambda \) satisfies Eq. (2). Then, every instance of every operation \( \lambda_t \in \lambda \) finishes its execution within its period of length \( T_k^a \), while guaranteeing the sum of power demands at any time instant is no larger than the battery power capability (i.e., \( S_1 \) and \( S_3 \) hold).

\[
\sum_{\lambda_t \in \lambda \setminus \{ \lambda_h \}} I_{k-i}(T_k^a - L_k^a + \epsilon_i) < (P_{\text{bat}}^a - P_k^a + \epsilon_p) \cdot (T_k^a - L_k^a + \epsilon_i),
\]

where \( I_{k-i}(\ell) = W_{k-i}(\ell) \), if \( \lambda_h \) has a lower-priority than \( \lambda_t \) or \( P_i^a < P_k^a \); \( I_{k-i}(\ell) = \min\left(P_i^a, P_{\text{bat}}^a - P_k^a + \epsilon_p\right) \cdot \min\left(L_i^a, \ell, T_i^a\right) \) otherwise.

\[
\text{Proof.}\quad \text{As discussed so far, } I_{k-i}(T_k^a - L_k^a + \epsilon_i) \text{ in Eq. (2) is an upper-bound of the amount of energy demanded by instances of } \lambda_t \text{ in an interval of length } (T_k^a - L_k^a + \epsilon_i). \text{ Therefore, if Eq. (2) holds, then there exists an instant } t_i \text{ within the interval, such that the sum of power demands is less than or equal to } (P_{\text{bat}}^a - P_k^a) \cdot (T_k^a - L_k^a) \text{ time units after its release, implying that } \lambda_h \text{ finishes its execution within its period.} \]

The lemma works not only for an offline power guarantee for the case of the sole supply, but also for a basis to develop an offline power guarantee for the general case to be discussed in Section 4. Also, the lemma can be used for addressing a design problem: calculation of the minimum capability of the battery cell that can supply given \( \lambda \) by finding the minimum \( P_{\text{bat}}^a \) that satisfies the lemma for given \( \lambda \).

### 3.3 Improved Power-Guarantee Analysis

We now develop a new power-guarantee analysis, which is an improved version of the one presented above. We first
Lemma 2. Suppose that every $\lambda_k \in \lambda$ satisfies Eq. (4), and $S_i$ for every $\lambda_k \in \lambda$ is given. Then, every instance of every operation $\lambda_k \in \lambda$ finishes its execution during its period of length $T_k$, while guaranteeing the sum of power demands at any time instant is no larger than the battery power capability (i.e., $S_1$ and $S_3$ hold).

\[
\sum_{\lambda_k \in \lambda \setminus \{\lambda\}} I_{k-1}(T_k^d - L_k^d + \epsilon_t, S_i) < (P_{\text{bat}}^d - P_1^d + \epsilon_p) \cdot (T_k^d - L_k^d + \epsilon_t),
\]

where $I_{k-1}(\ell, S_i) = W_{k-1}^*(\ell, S_i)$, if $\lambda_k$ has a lower-priority than $\lambda_1$ or $P_1^d < P_2^d$; $I_{k-1}(\ell) = \min(P_1^d, P_{\text{bat}}^d - P_2^d + \epsilon_p) \cdot \min(L_k^d - \epsilon_t, t)$ otherwise.

Proof. By the definition of $W_{k-1}^*(\ell, S_i)$, $I_{k-1}(T_k^d - L_k^d + \epsilon_t, S_i)$ in Eq. (4) is an upper-bound of the amount of energy demanded by instances of $\lambda_k$ in an interval of length $(T_k^d - L_k^d + \epsilon_t)$ when the slack of $\lambda_k$ is $S_i$. The remainder of the proof is the same as Lemma 1. \Box

For power guarantee, Lemma 2 misses one important issue—how to calculate $S_i$. Using Eq. (4), we can calculate $S_i$ as follows.

**Lemma 3.** The slack of $\lambda_k \in \lambda$ (denoted by $S_k$) can be lower-bounded as:

\[
S_k \geq T_k^d - L_k^d - \sum_{\lambda_k \in \lambda \setminus \{\lambda\}} I_{k-1}(T_k^d - L_k^d + \epsilon_t, S_i) \frac{P_{\text{bat}}^d - P_2^d + \epsilon_p}{P_{\text{bat}}^d}.
\]

**Proof.** The lemma holds by Eq. (4). That is, Eq. (4) can be rephrased as:

\[
\sum_{\lambda_k \in \lambda \setminus \{\lambda\}} I_{k-1}(T_k^d - L_k^d + \epsilon_t, S_i) \frac{P_{\text{bat}}^d - P_2^d + \epsilon_p}{P_{\text{bat}}^d} < T_k^d - L_k^d + \epsilon_t.
\]

Then, the physical meaning of the left-hand side of Eq. (6) is the maximum interval length that other tasks than $\lambda_k$ prevent $\lambda_k$ from execution. This means that the interval length between $\lambda_k$’s release and finishing times is at most as much as the sum of the left-hand side of Eq. (6) (i.e., interference from other operations than $\lambda_k$) and $L_k^d$ (i.e., the execution time of $\lambda_k$ itself). Therefore, the slack value of $\lambda_k$ is at least as much as the right-hand side of Eq. (5). \Box

The overall process on how to incorporate Lemma 3 into Lemma 2 is the same as the slack reclamation of real-time scheduling [33]. That is, in the first iteration, we apply Lemma 2 with $S_k = 0$ for every $\lambda_k \in \lambda$, and update $S_k > 0$ for every $\lambda_k \in \lambda$ using Lemma 3. In the second iteration, we repeat the same process with the new slacks. This process is repeated until every operation satisfies Eq. (4) (achieving the power guarantee) or there is no more slack update (failing the power guarantee).

In terms of power guarantee, it is trivial to show that the improved version (Lemma 2 with Lemma 3) always better performance than Lemma 1. That is because Lemma 2 with $S_k = 0$ for every $\lambda_k \in \lambda$ is equivalent to Lemma 1. On the other hand, the improved version incurs high time-complexity due to some iteration. Due to the iteration, the time-complexity of the slack reclamation is known to be pseudo-polynomial [33]. Since we are interested in “offline” power guarantees, the pseudo-polynomial time-complexity is acceptable for most cases. In Section 6, we will compare the performance of the vanilla (simple) and improved versions of the proposed offline power guarantee.

**4 SCHEDULING OF MULTIPLE POWER-DEMAND OPERATIONS WITH MULTIPLE POWER-SUPPLY SOURCES**

This section addresses a more general situation than Section 3, in which additional power is sporadically generated from multiple power sources such as an RBS and a solar panel, and immediately stored in the energy buffer or used for power-demand operations. We will first address a scheduling challenge due to the existence of sporadic additional power supply. Second, we present two approaches, depending on how to distribute the additional power supply to power-demand operations. Finally, we describe how to incorporate the two approaches in the proposed offline power-guarantee analysis.

**4.1 A Scheduling Challenge**

Unlike the situation where a battery pack is the only power-supply source discussed in Section 3, a straightforward
approach cannot yield a system-level power guarantee, as shown in the following example.

**Example 1.** Suppose that additional power is supplied by \( \Gamma_1 \) in \([t_1, t_2]\), while the battery pack is the only supply in \([t_0, t_1) \) and \([t_2, t_3]\), where \( t_0 < t_1 < t_2 < t_3 \), as shown in Fig. 17a in the supplement, available online. Also, there are two operations \( \lambda_1 \) and \( \lambda_2 \) ready to execute at \( t_0 \), and \( \lambda_1 \leq P_{\text{bat}} \) and \( \lambda_1 + \lambda_2 > P_{\text{bat}} \), as shown in Fig. 17b in the supplement, available online. Suppose that we apply the worst-case-based work-conserving policy in Algorithm 1, implying we start execution of an operation in the ready queue as long as the system has enough remaining power supply to accommodate the worst-case power demand of the operation. Then, \( \lambda_2 \) can start its execution at \( t_1 \), but there is a problem at \( t_2 \), at which power supplied by \( \Gamma_1 \) ends. In \([t_2, t_3]\), the total amount of power demand is strictly larger than that of power supply, entailing either eviction of one of “non-preemptive” operations, or risking power shortage in executing operations, both of which are considered as a system failure.

Example 1 shows the need for a more fine-grained way to handle additional sporadic power-supply sources. To meet this need, we consider two policies depending on how to distribute the additional supply as follows.

- Calculate the additional “guaranteed” uniform supply \( P_{\text{uni}}^a \), meaning that additional power-supply sources (and the energy buffer) can always provide power as much as \( P_{\text{uni}}^a \) (as the battery pack provides up to \( P_{\text{bat}}^a \)). This entails the calculation of \( P_{\text{uni}}^a \), once it is calculated, we can reuse the power scheduling and analysis framework presented in Section 3, by adding \( P_{\text{uni}}^a \) to the existing uniform supply \( P_{\text{bat}} \). In this case, all operations can share the power generated by additional power-supply sources.

- Assign additional power to a partial set of operations. Power generated by additional power-supply sources (and stored in the energy buffer) is used only when the operations in the partial set are executed. This entails the way to divide power generated by additional power-supply sources for individual power-demand operations.

In what follows, we will detail the above two approaches, including their scheduling frameworks.

### 4.2 Uniform Supply

In this approach, we calculate the additional guaranteed uniform supply \( P_{\text{uni}}^a \) from additional power-supply sources. After calculating \( P_{\text{uni}}^a \), we can reuse the scheduling framework in Algorithm 1. That is, we just change the \( P_{\text{bat}} \) term in Line 5 to \( P_{\text{bat}} + P_{\text{uni}}^a \). The main issue of this approach is to accurately calculate \( P_{\text{uni}}^a \); the larger \( P_{\text{uni}}^a \), the more operations to be accommodated.

The basic idea to obtain \( P_{\text{uni}}^a \) is to calculate the amount of the minimum supplied energy in \([0, t]\) by considering the fact that the energy buffer has at least \( P_{\text{UC}}^a \cdot L_{\text{UC}}^a \) of energy at 0 and each additional supply generates power at the end of its instances’ periods. If we divide this amount by \( t \) and take the minimum, we guarantee to supply power as much as \( P_{\text{uni}}^a \) in \([0, t]\), as stated in the following lemma.

**Lemma 4.** We can calculate \( P_{\text{uni}}^a \) using the following equation.

\[
P_{\text{uni}}^a = \min_{0 \leq t < L_{\text{LCM}}} \frac{f(\Gamma_{\text{UC}}, t) + \sum_{\Gamma_i \in \Gamma} f(\Gamma_i, t)}{t},
\]

where \( L_{\text{LCM}} \) is the least common multiple of \( \{T_i^a\}_{\Gamma_i \in \Gamma} \),

\[
f(\Gamma_{\text{UC}}, t) = P_{\text{UC}}^a \cdot \min(t, L_{\text{UC}}^a),
\]

\[
f(\Gamma_i, t) = \begin{cases} P_{\text{bat}}^a \cdot \left[ \frac{t}{T_i^a} \right] \cdot L_{i}^a & \\
+ P_{\text{bat}}^a \cdot \max\left(0, t - \left[ \frac{t}{T_i^a} \right] \cdot T_i^a - \left( T_i^a - L_i^a \right)\right) &
\end{cases}
\]

**Proof.** Since the amount of energy in the energy buffer at \( t = 0 \) is at least \( P_{\text{UC}}^a \cdot L_{\text{UC}}^a \), the amount of the supplied energy from the energy buffer in \([0, t]\) is \( P_{\text{UC}}^a \cdot \min(t, L_{\text{UC}}^a) \), and at least \( P_{\text{UC}}^a \cdot L_{\text{UC}}^a \) otherwise, which is recorded in \( f(\Gamma_{\text{UC}}, t) \) of Eq. (8). Given \( t \), \( \left[ \frac{t}{T_i^a} \right] \) means the number of instances of \( \lambda_i \) whose periods are completely included in \([0, t]\), and each instance generates energy no smaller than \( P_{\text{bat}}^a \cdot L_i^a \). The second term of Eq. (9) presents the minimum energy generated by the last instance whose period is partially included in \([0, t]\). Therefore, \( \sum_{\Gamma_i \in \Gamma} f(\Gamma_i, t) \) represents the amount of generated energy by \( \Gamma \) in \([0, t]\). Since we assume that the capacity of the energy buffer is sufficiently large, we can always use power as much as the lower-bound of \( \frac{f(\Gamma_{\text{UC}}, t) + \sum_{\Gamma_i \in \Gamma} f(\Gamma_i, t)}{t} \) for \( 0 \leq t < L_{\text{LCM}} \). \( \square \)

Note that the additional power supply should be stored in the energy buffer \( \Gamma_{\text{UC}} \), and hence \( P_{\text{uni}}^a \) cannot be larger than \( P_{\text{UC}}^a \).

If \( L_{\text{LCM}} \) is very large or time-complexity is critically important, e.g., for online admission control for operations, we need a tractable way to calculate \( P_{\text{uni}}^a \), which is covered in the following lemma.

**Lemma 5.** \( P_{\text{uni}}^a \) is at least as much as the minimum of \( P_{\text{UC}}^a \) and \( \sum_{\Gamma_i \in \Gamma'} P_{\text{uni}}^a \cdot L_i^a / T_i^a \) if \( \Gamma' \) satisfies Eq. (10).

\[
\sum_{\Gamma_i \in \Gamma'} (T_i^a - L_i^a) \cdot \frac{P_{\text{bat}}^a \cdot L_i^a}{T_i^a} \leq P_{\text{UC}}^a \cdot L_{\text{UC}}^a.
\]

**We will describe how to find \( \Gamma' \) after the lemma.**

**Proof.** Since \( \Gamma_i^a \) can supply power at most \( P_{\text{UC}}^a \), \( P_{\text{uni}}^a \) should be no larger than \( P_{\text{UC}}^a \). Otherwise, there should exist some \( \Gamma_i \) that always generates power, which is impossible because we cannot control if each \( \Gamma_i \) generates power.

While \( \frac{P_{\text{bat}}^a \cdot L_i^a}{T_i^a} \) is the average rate of power generation by \( \Gamma_i \), it cannot entirely contribute \( P_{\text{uni}}^a \). This is, because there is no power generation in \([0, T_i^a - L_i^a]\). To supply \( \frac{P_{\text{uni}}^a \cdot L_i^a}{T_i^a} \)-rate power in \([0, T_i^a - L_i^a]\), we need to use \( (T_i^a - L_i^a) \cdot \frac{P_{\text{uni}}^a \cdot L_i^a}{T_i^a} \) amount of power from \( \Gamma_{\text{UC}} \). Then, the generated energy from \( \Gamma_i \) in \([x \cdot T_i^a - L_i^a, x \cdot T_i^a]\) (as much as \( P_{\text{uni}}^a \cdot L_i^a \)) enables to supply \( \frac{P_{\text{uni}}^a \cdot L_i^a}{T_i^a} \)-rate power in \([x \cdot T_i^a - L_i^a, (x+1) \cdot T_i^a - L_i^a]\). Therefore, the LHS of Eq. (10) is an upper-bound of the amount of energy for every operation \( \lambda_i \in \Lambda' \) to contribute \( \frac{P_{\text{uni}}^a \cdot L_i^a}{T_i^a} \) to \( P_{\text{uni}}^a \).
Since the amount of power in $\Gamma_{uc}$ at $t = 0$ is $P_{0uc} \cdot L_{0uc}$, we enforce the constraint Eq. (10).

To calculate $P_{uni}^d$, using Lemma 5, we introduce a heuristic to find $\Gamma'$ that satisfies Eq. (10). Staring from $\Gamma' = \emptyset$, we repeat to add an operation in $\Gamma \setminus \Gamma'$ to $\Gamma'$ until Eq. (10) is satisfied. Here, the order to be selected from $\Gamma \setminus \Gamma'$ is based on the following criteria (the larger value, the earlier selection):

$$\frac{P_{uni}^d \cdot L_i^d}{T_i^d} = \frac{1}{T_i^d - L_i^d}. \quad (11)$$

We use the above criteria because a larger value of $\frac{1}{T_i - L_i}$ means more contribution of $P_{uni}^d$ (i.e., denominator) per usage of $P_{0uc} \cdot L_{0uc}$ (i.e., numerator).

We will describe how to use $P_{uni}^d$ in Section 4.4.

4.3 Dedicated Supply Approach

In this approach, we can determine $\lambda_{sup}$, a set of operations completely powered by a set of additional power-supply sources $\Gamma$ and the energy buffer $\Gamma_{uc}$. Once we determine $\lambda_{sup}$, operations in $\Gamma \setminus \lambda_{sup}$ can be executed according to Algorithm 1, and their power guarantee is judged by Lemma 1 with $\lambda_{sup}$. On the other hand, each operation in $\lambda_{sup}$ is fully supplied by $\Gamma$ with $\Gamma_{uc}$, and does not use power from $\Gamma_{bat}$. Our policy is to execute each operation in $\lambda_{sup}$ at the end of each period, which accommodates more operations in $\lambda_{sup}$ (because this policy uses less initial energy from the energy buffer). Formally, $\lambda_k \in \lambda_{sup}$ starts its execution at $r + T_k^d - L_k^d$, where $r$ is the release time of an instance of $\lambda_k$, and $\Gamma_{uc}$ supplies $P_{uni}^d (t) (\leq P_k^d)$ amount of power to $\lambda_k$ in $[r + T_k^d - L_k^d, r + T_k^d]$.

Then, the remaining step is to determine $\lambda_{sup}$. The basic idea is to calculate the maximum energy demanded by $\lambda_{sup}$ in $[0, t]$ and the minimum energy supplied by $\Gamma$ and $\Gamma_{uc}$ in $[0, t]$. We check whether the former is not larger than the latter at all times, which is stated in the following lemma.

Lemma 6. Every instance of every operation $\lambda_k \in \lambda_{sup}$ finishes its execution at the end of each period (e.g., $[r + T_k^d - L_k^d, r + T_k^d]$ where $r$ is the release time of an instance of $\lambda_k$) only with $\Gamma$ and $\Gamma_{uc}$, if the following inequality holds for all $t \in [0, \text{LCM})$.

$$\sum_{\lambda_j \in \lambda_{sup}} f(\lambda_i, t) \leq f(\Gamma_{uc}, t) + \sum_{\lambda_j \in \Gamma} f(\Gamma, t), \quad (12)$$

$$f(\lambda_i, t) = P_{uni}^d \cdot \left( \frac{t}{T_i^d} \cdot L_i^d \right)$$

$$+ P_{uni}^d \cdot \max \left( 0, t - \frac{t}{T_i^d} \cdot T_i^d - (T_i^d - L_i^d) \right). \quad (13)$$

Proof. Since $f(\lambda_i, t)$ in Eq. (13) exhibits the same formula as Eq. (9), it calculates the maximum energy demanded by $\lambda_i$ in $[0, t]$ when its instances are executed at the end of their periods. Therefore, the LHS (Left-Hand Side) of Eq. (12) is the maximum energy demanded by $\lambda_{sup}$ in $[0, t]$. On the other hand, the RHS (Right-Hand Side) of the equation is the minimum energy supplied by $\Gamma$ and $\Gamma_{uc}$ in $[0, t]$ as explained in Lemma 4. Therefore, the lemma follows.

If time-complexity is critical, we can use another necessary condition presented in the following lemma (likewise Lemma 5).

Lemma 7. Every instance of every operation $\lambda_k \in \lambda_{sup}$ finishes its power execution at the end of each period (i.e., $[r + T_k^d - L_k^d, r + T_k^d]$) where $r$ is the release time of an instance of $\lambda_k$, only with $\Gamma$ and $\Gamma_{uc}$ (without $\Gamma_{bat}$), if the following inequality holds:

$$\sum_{\lambda_j \in \lambda_{sup}} P_{uni}^d \cdot L_i^d \leq P_{uni}^d \text{in Lemma 4 (or Lemma 5).} \quad (14)$$

Proof. By the meaning of $P_{uni}^d$, $\Gamma$ and $\Gamma_{uc}$ can always supply $P_{uni}^d$-rate power. If we enforce each operation $\lambda_k \in \lambda_{sup}$ executes its instances at the end of their periods, their cumulative demand is always not larger than $P_{uni}^d$-rate power. Therefore, as long as their total demanded power rate (i.e., the LHS of Eq. (14)) is no larger than $P_{uni}^d$, every instance of every operation $\lambda_k \in \lambda_{sup}$ finishes its execution within its period of length $T_k^d$, only with $\Gamma$ and $\Gamma_{uc}$.

The remaining problem is then how to select $\lambda_{sup}$ that satisfies Lemma 6 (or Lemma 7). Here we describe a simple, but effective heuristic. We sort $\lambda_k \in \lambda$, based on the ratio of the LHS to the RHS of Eq. (2). If the ratio of $\lambda_i$ is larger than that of $\lambda_j$, we interpret that $\lambda_i$ is more difficult to satisfy Eq. (2) than $\lambda_j$. Therefore, starting from $\lambda_{sup} = \emptyset$, we repeat the following step until there is no operation to be moved: we select an operation $\lambda_j$ with the largest ratio among operations in $\lambda \setminus \lambda_{sup}$ such that $\lambda_{sup} \cup \{ \lambda_j \}$ satisfies Lemma 6 (or Lemma 7), and then add $\lambda_j$ to $\lambda_{sup}$.

Another simple strategy for selecting $\lambda_{sup}$ is the lower-priority-first strategy. That is, as we use fixed-priority scheduling, the lower-priority task tends to violate Eq. (2) (as well as Eq. (4)). Starting from $\lambda_{sup} = \emptyset$, we scan all $\lambda_k \in \lambda$ from the lowest-priority to highest-priority operations, and check Lemma 6 (or Lemma 7) is satisfied if $\lambda_k$ is included in $\lambda_{sup}$. If the lemma satisfied, we add $\lambda_k$ to $\lambda_{sup}$, otherwise, we do not add.

4.4 Incorporating Additional Supply to the Proposed Analysis

Section 4.2 calculates $P_{uni}^d$, the additional guaranteed uniform supply by additional power-supply sources, and Section 4.3 calculates $\lambda_{sup}$, a set of operations completely powered by a set of additional power-supply sources. The uniform and dedicated approaches utilize $P_{uni}^d$ and $\lambda_{sup}$, respectively, and improve the offline power guarantee as follows.

First, when it comes to the uniform supply, we can check a power guarantee of this approach by applying Lemma 1 (or Lemmas 2 and 3) for all $\lambda_i \in \lambda$ and replacing $P_{bat}^d$ with $P_{uni}^d + P_{uni}^d$. Second, as to the dedicated supply, we can check the power guarantee by checking Lemma 1 (or Lemma 2) only with $\lambda_i \in \lambda \setminus \lambda_{sup}$. Note that for $\lambda_i \in \lambda_{sup}$, we automatically guarantee their power sufficiency in that $\lambda_{sup}$ is constructed so as to supply all power demands in $\lambda_{sup}$ by $\Gamma$ and $\Gamma_{uc}$ without $\Gamma_{bat}$.\[\]
The amount of energy at (denoted by $t_0$ static-demand-based online power management amount of supplied by $X$ is a constant representing of any task. Then, the energy buffer supplies, not of $i$ $X$ by monitoring [15]. Lines 1 and 2 of Algorithm 2 to store based on their maximum power if at is a power consumption during a sub-period $t_i$ $P_{bu}$ period (for online power management) of length $t_p$. late the amount of energy in $1790$ IEEE TRANSACTIONS ON PARALLEL AND DISTRIBUTED SYSTEMS, VOL. 31, NO. 8, AUGUST 2020 work periodically controls different approaches to determine the battery current bound. The first approach (called static-demand-based online power management) assumes that we can predict future scheduling information of power-demanding tasks and use the information to determine the limits as shown in Fig. 2. For the situation where future power-demand information is not available, the second approach (called predicted-demand-based online power management) uses past power requirement information only as seen in Fig. 3; it keeps calculating average and standard deviation of the past power requirement, and determines the battery current bound.

5 ONLINE POWER MANAGEMENT

Thus far, we have explored an offline analysis. The offline analysis provides the system-level power guarantee in our scheduling framework based on power-demand and supply task information. We have also described how the sporadic power supplies can improve the power-guarantee analysis. However, if one performs a set of power-demand operations with a set of power-supply sources that satisfy the offline power guarantee analysis in Section 4, there will be extra energy stored in the energy buffer as the analysis is based on the minimum (not actual) power supply. Thus, we propose an online power management framework, which adaptively controls the power of the energy buffer based on scheduling information. The goal of the online management is to reduce the peak power supplied by the battery while guaranteeing system-power capability that achieves the goal in Section 2.2, the online power management framework determines a desirable battery current bound to achieve the peak power reduction. Then, whenever battery current exceeds the current bound, the system controls the energy buffer to supply the stored energy to the system load so as to limit the battery current. The energy system must be able to supply the required power within its system configurations and energy buffer states.

In this section, we have explored two different approaches to determine the battery current bound. The first approach (called static-demand-based online power management) assumes that we can predict future scheduling information of power-demanding tasks and use the information to determine the limits as shown in Fig. 2. For the situation where future power-demand information is not available, the second approach (called predicted-demand-based online power management) uses past power requirement information only as seen in Fig. 3; it keeps calculating average and standard deviation of the past power requirement, and determines the battery current bound.

5.1 Static-Demand-Based Online Power Management

The static-demand-based online power management framework periodically controls power of the energy buffer, as shown in Fig. 2 and Algorithm 2. At $t_0$, the beginning of each period (for online power management) of length $t_p$, we calculate the amount of energy in $G_{uc}$, which is necessary for an offline power guarantee for the current period $(t_0, t_0 + t_p)$ (denoted by $E_{buf}^{\text{uni}}(t_0)$) as follows. For the uniform power-supply approach in Section 4.2, $E_{buf}^{uni}(t_0)$ is simply calculated by $t_p \cdot P_{uni}^{uni}$ since the offline power guarantee exploits the property that $P_{uni}^{uni}$ of power is always supplied by $\Gamma$. For the dedicated supply approach in Section 4.3, we calculate $E_{buf}^{\text{uni}}(t_0)$ using the amount of energy consumed by $\lambda^{\text{uni}}$ based on their maximum power demand parameters (i.e., $P_{\text{uni}}^{\text{uni}}$, not $P_i^{\text{uni}}(t)$) during the interval.

Once $E_{buf}^{\text{uni}}(t_0)$ is calculated, we can utilize the energy from $G_{uc}$ up to the difference between the amount of total energy stored in $G_{uc}$ at $t_0$ (denoted by $E_{buf}(t_0)$) and $E_{buf}^{\text{uni}}(t_0)$. Note that we can measure the amount of energy in $G_{uc}$ by monitoring the voltage level of $G_{uc}$ at $t_0$ (denoted by $V_{buf}(t_0)$), using $E_{buf}(t_0) = \frac{1}{2} \cdot C_{buf} \cdot V_{buf}^2(t_0)$, where $C_{buf}$ is a constant representing the capacitance of $G_{uc}$ [15]. Lines 1 and 2 of Algorithm 2 represent the calculation of $E_{buf}^{\text{uni}}(t_0)$ and $E_{buf}(t_0)$.

Then, we utilize the extra energy up to as much as $E_{buf}(t_0) - E_{buf}^{\text{uni}}(t_0)$, for reducing the battery’s peak power. We use energy from the energy buffer if the power usage of the battery pack is larger than a threshold $I_{\text{bat}}(t_0)$. We determine $P_{\text{bat}}^{\text{uni}}(t_0)$ for the current period $(t_0, t_0 + t_p)$, using the scheduling results of the previous period $(t_0 - t_p, t_0)$, which is the worst-case energy consumption by $\lambda$ during that period. During each scheduling period, we can figure out how much can be stored in a buffer by $P_{\text{bat}}^{\text{uni}}(t_0)$ as follows.

$$f_{\text{FFP}}(P_s) = \begin{cases} -t_i \cdot P_s + t_i \cdot P_i, & \text{if } P_{\text{bat}}^{\text{uni}} < P_i, \\ 0, & \text{otherwise,} \end{cases}$$

where $P_i$ is a power consumption during a sub-period $t_i \in [t_0 - t_p, t_0)$ as seen in Fig. 4. Note that $P_i$ and $t_i$ are not typically the same as $P$ and $t$ of any task.

Then, sum of $f_{\text{FFP}}(P_s)$ represents total stored energy in the buffer because of the power threshold $(P_{\text{bat}}^{\text{uni}})$ as shown in Fig. 4. Next, $f_{\text{FFP}}(E)$, the function which finds $P_{\text{bat}}^{\text{uni}}$ to store the amount of buffer energy ($E$), can be derived by applying the inverse function of $\sum_{i=1}^{N} f_{\text{FFP}}(P_s)$. Finally, for given $E_{buf}^{\text{uni}}(t_0)$ and $P_{\text{bat}}^{\text{uni}}(t_0)$, the online power management framework controls the energy stored in the energy buffer within a period $(t_0, t_0 + t_p)$ as follows. Suppose that the battery pack should supply $X$ amount of power if there is no supply from $G_{uc}$ at $t$ for the purpose of the peak power reduction. Then, the energy buffer supplies $X - P_{\text{bat}}^{\text{uni}}(t_0)$ of power, only when $X \geq P_{\text{bat}}^{\text{uni}}(t_0)$ and $E_{buf}(t) \geq E_{buf}^{\text{uni}}(t_0)$ hold.

3. The amount of energy $G_{uc}$ (the energy buffer) should supply at $t$ for an offline power guarantee already figured in $X$. 

Fig. 2. Static-demand-based online power management: Addition of buffered energy can be calculated based on buffer voltage. Future power demand scheduling information and the buffered energy amount are used to determine the buffer power.

Fig. 3. Predicted-demand-based online power management: Power demand history and the buffered energy amount are used to determine the buffer power.
Algorithm 2. Static-Demand-Based Online Power Management

The following steps are performed at \( t_0 \), the beginning of each period of length \( T_p \).

1. Calculate \( E_{\text{net}}(t_0) \) depending on the uniform/dedicated supply approach.
2. Calculate \( E_{\text{out}}(t_0) \) by \( \frac{1}{2} \cdot C_{\text{out}} \cdot V_{\text{out}}^2(t_0) \).
3. \( P_{\text{bat}}(t_0) \leftarrow f_{\text{DSP}}(E_{\text{net}}(t_0)) \)

5.2 Predicted-Demand-Based Online Power Management

The goal of online power management is to minimize the battery (dis)charge stress by determining effective (dis)charge current limits in real time. The predicted-demand-based approach to be described in this subsection determines the limits based on the past power requirements and the online buffer energy state. The normal distribution of power requirements has been used to identify the past power requirements patterns. We adapt the formulations and ideas of the authors’ recent conference paper [34] for this approach. We first formally state the problem as follows.

Given the power requirement \( (I_{\text{req}}(t)) \), determine the (dis)charge currents from the energy buffer \( I_L(t) \), such that

\[
\text{Minimize (S4)} \quad \text{Discharge/charge stress} = \frac{1}{T_{op}} \int a \cdot e^{b |I_s(t)|} \, dt,
\]

Subject to (S5) \( I_{\text{req}}(t) = I_L(t) + I_{\text{std}}(t) \).

(15)

where \( I_L \) is a buffer current, \( a \) and \( b \) are the coefficients in the battery stress model. The energy storage system must supply the required power with current \( I_{\text{req}} \) to the electric load. As energy conserves, the sum of current supplies from energy storage components \( (I_L, I_{\text{std}}) \) must be the power requirement for all \( k \in [0, k_c] \) (S5). The objective function in (S4) captures the (dis)charge stress during an operation period of \( T_{op} \).

In this paper, the battery stress is an external condition and electrochemical reaction that affects battery degradation and lifetime [35], [36]. This work utilizes the current degradation model to evaluate the impact of discharge/charge current management on degradation and lifetime, because power buffer can control discharge/charge current and degradation rate. We assume the (dis)charge stress to be exponential with the (dis)charge current, because large (dis)charge current exponentially accelerates the battery degradation by exciting high over-potential between solid-phase and solution-phase voltages [36].

Algorithm 3. Target Current Decision \( (I_L) \)

1: \( \text{while } \{ \text{do} \)
2: \( \text{if } I_{\text{ub}} < I_{\text{req}} \text{ then } /* \text{Peak power reduction } */ \)
3: \( I_L \leftarrow I_L - I_{\text{ub}} \)
4: \( \text{else} \)
5: \( I_L \leftarrow 0 \)
6: \( \text{end if} \)
7: \( \text{end while} \)

Algorithm 3 and Figs. 5 and 6 show the overall algorithm. Lines 2–6 of Algorithm 3 determine the discharge current from the power-dense battery \( I_L \) to meet the high (dis)charge current requirement \( I_{\text{req}} \). When \( I_{\text{req}} \) is greater than its upper bound \( I_{\text{ub}} \), the power-dense battery provides energy to reduce the discharge current of the energy-dense battery (Lines 2–3). For the effective power buffering, current upper bounds are critical because they dictate the amount of discharge current of the storage.

Now we describe how to determine the current bounds \( (I_{\text{ub}}, I_{\text{std}}) \). Algorithm 4 explains the main current bound searching algorithm and its preparation steps. The first preparation step is to calculate the available energy in the buffer \( (E_c) \). If we use an ultra-capacitor (UC) as the power-dense energy buffer, we can use the standard equation related to the capacitor energy \( E_c = \frac{1}{2} C_c V_{c,\text{rated}}^2 \), where \( V_{c,\text{rated}} \) denotes the UC-rated voltage and \( C_c \) denotes the UC capacitance.
Next, it statistically figures out discharge/charge pattern including average charging/recharging cycle period ($T_c$), average current ($I_{avg}$) and its standard deviation ($I_{std}$). Then, the current bound is determined under the assumption that the required current would follow the normal distribution with average ($I_{avg}$) and standard deviation ($I_{std}$), over the next control period. When the average required current ($I_{avg}$) is high, the current bound $I_{ub}$ should be increased to effectively buffer the recharging energy while reducing the charge stress. This buffered energy will be supplied to the load to mitigate the energy-dense battery discharge current as shown in Fig. 5. The standard deviation of the required current ($I_{std}$) also affects the determination of $I_{ub}$. A large standard deviation means the transfer of a large amount of energy with a high peak current. In such a case, $I_{ub}$ should be increased to buffer energy effectively without incurring shortage of energy in the power-dense energy buffer as shown in Fig. 5.

Algorithm 5 and Fig. 6 describe how to determine $I_{ub}$ based on the energy capacity of the buffer and the history of required power. To maximize the utilization of power-dense energy buffer, it should spend most energy in the buffer during the discharging period while minimizing peak discharge current ($I_{ub}$). This means $I_{ub}$ should not only be set to a small value as possible, but also make the supplied energy from the buffer ($E_{ub}$) equal to the stored energy in the buffer ($E_c$) under the load requirement pattern. In the algorithm, it searches proper $I_{ub}$ meeting these conditions. For each case of $I_{ub}$, it calculates the supplied energy by integrating buffered power ($P_{ub} = V_b I_{ub} = V_b(i - I_{ub})$) during one charging/recharging cycle ($T_c$) as:

$$E_{ub} = \int_0^{T_c} P_{ub}(t) dt = T_c \int_{I_{ub}}^\infty P_{ub}(i) PDF(i) di$$

$$= T_c \int_{I_{ub}}^\infty V_b I_{ub}(i) PDF(i) di$$

$$= T_c \int_{I_{ub}}^\infty V_b(i - I_{ub}) PDF(i) di$$

$$= T_c V_b \int_{I_{ub}}^\infty \int_{I_{ub}}^\infty iPDF(i) di - I_{ub} \int_{I_{ub}}^\infty PDF(i) di$$

$$= T_c V_b \left[ \int_{I_{ub}}^\infty iPDF(i) di - I_{ub} \int_{I_{ub}}^\infty PDF(i) di \right]$$

$$= T_c V_b \left[ \int_{I_{ub}}^\infty iPDF(i) di - I_{ub}(1 - CDF(I_{ub})) \right]$$

$$= T_c V_b \left[ \int_{I_{ub}}^\infty PDF(i) di + (I_{avg} - I_{ub})(1 - CDF(I_{ub})) \right],$$

(16)

where PDF($i$) (likewise CDF($i$)) is a probability (likewise cumulative) density function of a current $i$, $V_b$ is the battery output voltage, and $P_{ub}$ (likewise $I_{ub}$) is the buffered power (likewise current). In the first line, we transform the buffered power over time, $P_{ub}(t)$, to the buffered power over current, $P_{ub}(i)$.

Thus, we can statistically calculate the expected buffered energy based on a probability density function of a current, PDF($i$), and buffered current ($I_{ub}$) as shown in the second line. We assumed battery voltage and system output voltage is the same and constant, so the buffered power can be calculated by multiplying the voltage ($V_b$) and current flowing through the energy buffer ($I_{ub} = i - I_{ub}$) as described in the third line. After rearranging the terms in the equation, we can derive the equation calculating the amount of the buffered energy which is function of $I_{ub}$. Note that for a normal distribution, we can calculate $\int_{I_{ub}}^\infty iPDF(i) di$ in Eq. (16); see Eq. (17) in the supplement, available online.

6 Evaluation of Offline Power-Supply Guarantee Analysis

In this section, we demonstrate the performance of the offline power-supply guarantee analysis presented in Sections 3 and 4. First, we evaluate the analysis only with a uniform supply in Section 3, and then that with multiple power-supply sources in Section 4.
6.1 Results Without Additional Power-Supply Sources

To evaluate the offline power-supply guarantee analysis with a uniform supply, we need to determine parameters of the battery pack and a set of power-demand operations. For the power capability of the battery pack, we follow the parameters of the case study to be described in Section 7, setting $P_{bat}$ to 40W (see Table 1). For each power-demand operation $\lambda_k$, we determine its parameters $T^k_d$, $L^k_d$ and $P^k_d$ as follows. $T^k_d$ is uniformly chosen in $[1.0, 10.0]$s; $L^k_d$ is uniformly distributed in $[0.1, 0.5 \cdot T^k_d]$s; and $P^k_d$ is uniformly chosen in $[5.0, 15.0]$W. We also set $\epsilon_i$ and $\epsilon_p$ to 0.1s and 0.1W. Note that the scales of the above parameters are determined by considering the parameters of power-demand operations in the case study to be described in Section 7 (see Table 2).

We consider 10 choices of $n$ (1, 2, 3, ..., 10) for the number of operations in each set of power-demand operations. We randomly generate 10,000 operation sets for each $n$ (a total of 100,000 operation sets), and check the ratio of operation sets power-guaranteed by:

- **Plain–None**: the plain analysis (i.e., Lemma 1), and
- **Improved–None**: the improved analysis (i.e., Lemmas 2 with Lemma 3).

We would like to mention that the guarantee ratio shown in the Y-axis in Fig. 7 means the ratio of operation sets, which are power-guaranteed by the corresponding analysis and framework. For example, in Fig. 7a, the dotted and solid lines for the X-axis of 4 (i.e., the number of operations) correspond to 78.30 and 88.16 percent, respectively. This means that out of 10,000 operation sets, 7,830 and 8,816 operation sets are

---

**TABLE 1**
Parameters of the Power Supply: Renewable Energy Sources $\Gamma_1-\Gamma_3$, the Energy Buffer $\Gamma_{UC}$, and the Battery Pack $\Gamma_{bat}$

<table>
<thead>
<tr>
<th>Tasks</th>
<th>$T^s(s)$</th>
<th>$L^s(s)$</th>
<th>$P^s(W)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_1$</td>
<td>6.0</td>
<td>2.0</td>
<td>3.0</td>
</tr>
<tr>
<td>$\Gamma_2$</td>
<td>6.0</td>
<td>1.5</td>
<td>3.0</td>
</tr>
<tr>
<td>$\Gamma_3$</td>
<td>1.5</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>$\Gamma_{UC}$</td>
<td>3.0</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>$\Gamma_{bat}$</td>
<td>40.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 2**
Parameters of the Power-Demand Operations: $\lambda_1-\lambda_5$

<table>
<thead>
<tr>
<th>Tasks</th>
<th>$T^d(s)$</th>
<th>$L^d(s)$</th>
<th>$P^d(W)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>6.0</td>
<td>2.0</td>
<td>12.0</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>6.0</td>
<td>2.0</td>
<td>9.6</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>4.0</td>
<td>2.0</td>
<td>6.0</td>
</tr>
<tr>
<td>$\lambda_4$</td>
<td>4.0</td>
<td>1.0</td>
<td>6.0</td>
</tr>
<tr>
<td>$\lambda_5$</td>
<td>6.0</td>
<td>4.0</td>
<td>7.2</td>
</tr>
</tbody>
</table>

---

Fig. 7. The ratio of operation sets power-guaranteed under various situations.
power-guaranteed by Plain—None and Improved—None, respectively, when the X-axis is 4.

We have two observations on the results shown in Fig. 7a. First, both Plain—None and Improved—None guarantee the power supply of all operation sets when \( n = 1 \) or \( n = 2 \); the ratio of operation sets guaranteed decreases as \( n \) increases, and finally, the ratio becomes almost zero when \( n = 10 \). Second, Improved—None always results in no smaller ratio than Plain—None. The difference between the two ratios is maximized when \( n = 5 \) (57.22 percent versus 37.73 percent), which demonstrates the effectiveness of the improved analysis compared to the plain analysis.

### 6.2 Results With Multiple Power-Supply Sources

We now evaluate the offline power-supply guarantee analysis with multiple power-supply sources. Similar to the previous subsection, we need to determine parameters of not only power-supply sources, but also power-demand operations. For power-supply sources, we also follow the parameters of the case study to be described in Section 7. That is, we set \( P_{bat}^{A}, P_{bat}^{B}, P_{bat}^{C} \), and \( L_{bat}^{A}, L_{bat}^{B}, L_{bat}^{C} \) to 40W, 3W and 3s, respectively, and \( \lambda_1, \lambda_2 \) and \( \lambda_3 \) have the parameters (6,2,3), (6,1.5,2.3) and (1,5,0,2,0,2), respectively, where \( (A, B, C) \) implies \( T^a = A(s), L^a = B(s) \) and \( I^a = C(W) \), as shown in Table 1 for the case study to be described. We consider two situations for power-supply equipment in addition to \( \Gamma_{bat} \): (i) the system equipped with \( \Gamma_1 \), and (ii) the system equipped with \( \Gamma_2 \) and \( \Gamma_3 \). We will use the same operation sets as the previous section (i.e., 10,000 sets for each \( n \)), and check the ratio of operation sets guaranteed by the followings:\(^5\)

- **Plain—Uni—i**: the uniform supply (Lemma 4) with the plain analysis (Lemma 1) when the additional power-supply is \( \Gamma_1 \),
- **Improved—Uni—i**: the uniform supply with the improved analysis (i.e., Lemmas 2 with Lemma 3) when the additional power-supply is \( \Gamma_1 \),
- **Plain—Ded—i**: the dedicated supply (Lemma 6) with the plain analysis when the additional power-supply is \( \Gamma_1 \),
- **Improved—Ded—i**: the dedicated supply with the improved analysis when the additional power-supply is \( \Gamma_1 \),
- **Plain—Uni—ii**: the uniform supply with the plain analysis when the additional power-supplies are \( \Gamma_1, \Gamma_2 \) and \( \Gamma_3 \),
- **Improved—Uni—ii**: the uniform supply with the improved analysis when the additional power-supplies are \( \Gamma_1, \Gamma_2 \) and \( \Gamma_3 \),
- **Plain—Ded—ii**: the dedicated supply with the plain analysis when the additional power-supplies are \( \Gamma_1, \Gamma_2 \) and \( \Gamma_3 \), and
- **Improved—Ded—ii**: the dedicated supply with the improved analysis when the additional power-supplies are \( \Gamma_1, \Gamma_2 \) and \( \Gamma_3 \).

We can summarize the results shown in Figs. 7b, 7c, 7d, 7e, 7f, 7g, 7h, and 7i as follows.

**R1.** The ratio by both Improved—X—Y and Plain—X—Y decreases as \( n \) increases, from 100 percent with \( n = 1 \) to near-zero with \( n = 10 \) as shown in all the figures.

**R2.** Improved—X—Y always results in no smaller ratio than Plain—X—Y, but the amount of improvement of Improved—X—Y over Plain—X—Y varies with the approach (uniform versus dedicated supply) and additional power-supply sources (\( \Gamma_1 \) versus \( \Gamma_2 \) and \( \Gamma_3 \)); see Figs. 7b, 7c, 7d, and 7e.

**R3.** The dedicated supply (i.e., X—Ded—Y) exhibits better performance than the uniform supply (i.e., X—Uni—Y) as shown in Figs. 7f, 7g, 7h, and 7i.

**R4.** The gap between ratios by X—Ded—Y and X—Uni—Y when X—Improved is smaller than the gap when X—Plain; see Figs. 7f, 7g, 7h, and 7i.

R1 and R2 are straightforward. For R2, the maximum differences between the ratio by Improved—Y—Z and that by Plain—Y—Z for each pair of Y and Z are 19.21 percent (= 61.3% − 42.09%) with \( n = 5 \) when \( Y = \text{Uni} \) and \( Z = \text{i} \), 18.92 percent (= 64.23% − 45.31%) with \( n = 5 \) when \( Y = \text{Ded} \) and \( Z = \text{i} \), 13.73 percent (= 37.64% − 23.91%) with \( n = 6 \) when \( Y = \text{Uni} \) and \( Z = \text{ii} \), and 12.74 percent (= 48.33% − 35.59%) with \( n = 6 \) when \( Y = \text{Ded} \) and \( Z = \text{ii} \).

On the other hand, R3 and R4 are non-trivial. Although the ratio by X—Ded—Z seems higher than the ratio by X—Uni—Z in every situation, X—Ded—Z cannot dominate X—Uni—Z,\(^6\) which is different from the dominant relation of Improved—Y—Z over Plain—Y—Z. That is, there exist many operation sets which are power-guaranteed by X—Uni—Y but not by X—Ded—Z; for example, there exist 1.12, 1.56, 0.53 and 0.79 percent such operation sets with \( n = 5 \) when the pair of (X and Z) is (Plain, i), (Improved, i), (Plain, ii) and (Improved, ii), respectively. Therefore, X—Uni—Z complements X—Ded—Z in terms of offline power guarantee. Also, we can observe R4. The gap between ratio by X—Ded—i and X—Uni—i with \( n = 5 \) is 12.15 percent (= 54.24% − 42.09%) when X—Plain, but the gap is reduced to 4.99 percent (= 66.29% − 61.31%) when X—Improved. Likewise, the same holds between the ratio by X—Ded—ii and by X—Uni—ii: 21.8 percent when X—Plain and 11.66 percent when X—Improved. This is because the improved analysis does not have much room for improvement since it already improved the power guarantee over the plain analysis.

### 7 Case Study for the Proposed Framework

We now conduct a case study and demonstrate how much real-world systems can benefit from the proposed framework. It consists of the offline power-supply guarantee analysis (in Sections 3 and 4) and online power management (in Section 5). We focus on whether or not they meet the goals stated in Section 2.2. To this end, we first build a simulation model on Matlab Simulink and run simulations with the proposed algorithms in the model, in order to demonstrate our framework guarantees power-supply requirement and reduces the energy dissipation. Hardware and parameters in the case study are derived from the prototype consisting of HESS developed in our earlier conference version [30]. We then present experimental results for the simulation.
model, demonstrating the HESS’s power-supply guarantee and reduction of energy dissipation.

7.1 Case Study With the Simulation Model
We target an actual system as a case study, which is equipped with wheels, wheel motors, stepper motors, coolers and a HESS including a pack of lithium-ion batteries, a pack of UCs, an RBS and solar panels; the actual system comes from the prototype developed in our preliminary conference version [30]. We can determine the parameters of power-demand operations and power-supply sources from their power demand/supply profiles and specifications. Specifications of energy sources/storage are presented in Table 1, and those of the power-demand operations are listed in Table 2. Based on the parameters, we determine the minimum battery capacity achieving a power-supply guarantee for the system via the power guarantee analysis in Sections 3 and 4. We have then executed various sequences of operations while recording battery states to evaluate the proposed system. The applications and tasks considered in the case study are detailed as follows.

Our driving application is programmed to operate wheel motors to achieve the driving profile. Our motor control \( \lambda_1 \) depends on the PID controller to achieve the required speed, its control interval \( (T^p_d) \), maximum acceleration time (time to achieve the target speed, \( L^d_1 \)), and the maximum power \( (P^d_d) \) are 6s, 2s, and 12W, respectively. To reflect various user applications, we also ran applications that sporadically actuate motors \( \lambda_2 \) and \( \lambda_3 \). Some applications actuate stepper motors to control position \( \lambda_4 \), while others \( \lambda_5 \) use thermal fins to regulate temperature for system thermal stability. The power-demand operations are shown in Table 2.

We deployed two types of solar panels. One solar panel can generate 10W, and the other 25W at sunny day. Its performance dropped largely at cloudy. So, we assumed the minimum power rate would be 3W during 1.5 seconds for the small panel and 3W during 2 seconds for the large one every 6 seconds at our application environment \( (\Gamma_1 \text{ and } \Gamma_2) \). We also assumed 0.2W of power is regenerated for 0.2 seconds every 1.5 seconds during driving application \( \Gamma_3 \). We used an ultra capacitor pack for the energy buffer which has 400F capacitance and 2.7V of rated-voltage. We designed a switched mode DC/DC converter to move energy between the UCs and battery cells. For this experiments, we optimized the converter configuration to maximize the converter efficiency at 3W power transfer for 3 seconds \( (\Gamma_{\text{UC}}) \). We targeted lithium-ion batteries each of which has 1400mAh of energy capacity and 4.2W of the minimum power capability, under the assumption that each cell has 3.0V minimum voltage and 1C-rate of power rate. We have to determine the number of battery cells (which is equivalent to the battery capacity \( \Gamma_{\text{bat}} \)) to guarantee power capability for the power-demand operations.

When it comes to the simulation model for the case study, we utilize Matlab Simulink standard with the Simscape model library to simulate the architecture as shown in Fig. 18 in the supplement, available online. The boost DC-DC converter model is used to connect UC and battery. We extracted two RC battery parameters from battery cells which is used in the case study. We used RC parameters of the UC that is described in its data sheet [37].

7.2 Evaluation for the Case Study
In this subsection, we evaluate our offline power-supply guarantee analysis and online power management for the case study.

7.2.1 Offline Power-Supply Guarantee Analysis
As mentioned in the description of the applications and tasks in the case study, we schedule the five operations in Table 2 supplied by the power sources in Table 1. We compare the three frameworks associated with the plain offline power guarantee analysis in Section 3.2 (i.e., Lemma 1, denoted by Plain) and its improved version in Section 3.3 (i.e., Lemma 2 with Lemma 3, denoted by Improved) as follows: (a) the framework without additional supply sources (using \( \Gamma_{\text{bat}} \) only), (b) the framework with additional supply sources (using all the supply sources in Table 1), which operate as a uniform supply, and (c) the framework with additional supply sources (using all the supply sources in Table 1), which operate as a dedicated supply. We use the same notations explained in Section 6 as follows:

- Plain—None: (a) associated with the plain analysis,
- Improved—None: (a) associated with the improved analysis,
- Plain—Uni—ii: (b) associated with the plain analysis,
- Improved—Uni—ii: (b) associated with the improved analysis,
- Plain—Ded—ii: (c) associated with the plain analysis, and
- Improved—Ded—ii: (c) associated with the improved analysis.

Note that we use the same notations as Section 6.

If we apply Plain—None, Lemma 1 cannot guarantee the timely execution of \( \lambda_1 \) (as well as \( \lambda_2 \)) with power requirement. That is, during the interval of length \( (T^d_2 - L^d_2 + \epsilon_i) = 6.0 - 4.0 + 0.1 = 2.1, \sum_{\lambda_i \in \{\lambda_1, \lambda_2, \lambda_3\}} I_{\text{sum}}(T^d_2 - L^d_2 + \epsilon_i) = 2.1 \cdot 12.0 + 2.1 \cdot 9.6 + 2.1 \cdot 6.0 + 2.1 \cdot 6.0 = 2.1 \cdot 33.6 = 70.56, \) which is no less than \( (P^a_{\text{bat}} - P^a_{\text{d}} + \epsilon_i) \cdot (T^d_2 - L^d_2 + \epsilon_i) = (40.0 - 7.2 + 0.1) \cdot (3.0 - 1.0 + 0.1) = 2.1 \cdot 21 = 69.09.\) However, if we apply Improved—None, Lemma 2 with Lemma 3 guarantees that every power demand operation in Table 2 can finish its execution within its deadline, without experiencing power deficiency. That is, after three iterations for finding slack values in Lemma 3, the slack values of \( \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \) are 1.6, 1.6, 0.1, 0.7, 0.0, respectively; then, during the interval of length \( (T^d_2 - L^d_2 + \epsilon_i) = 6.0 - 4.0 + 0.1 = 2.1, \sum_{\lambda_i \in \{\lambda_1, \lambda_2, \lambda_3\}} I_{\text{sum}}(2.1) = 2.1 \cdot 1.4 \cdot 6.0 = 63.6, \) which is less than 69.09.

We now explain how Improved—None calculates proper power requirement, compared to MAX and AVG, which denote the sum of the maximum power demand (i.e., \( \sum_{\lambda_i \in \{\lambda_1, \lambda_2, \lambda_3\}} P^a_{\text{max}} = 40.8W \)), and the average of the maximum power demand (i.e., \( \sum_{\lambda_i \in \{\lambda_1, \lambda_2, \lambda_3\}} P^a_{\text{avg}} = 16.5W \)). First, we can find the lowest battery capacity \( \Gamma_{\text{bat}} \) that makes Improved—None give power supply guarantee, which is \( \Gamma_{\text{bat}} = 38.0W. \) Then, Improved—None (38.0W) reduces the required battery capability. Also, it is straightforward that, if the battery

7. We can find such battery capacity by applying binary search.
capacity is as much as AVG, we cannot supply the sufficient power in the worst case.

When it comes to Improved–Uni–ii and Improved–Ded – ii, they further reduce the required battery capability by consuming 36.3W and 34.8W, respectively.\(^8\) Between the two, Improved–Ded – ii is the most effective framework for a power guarantee, in that it can reduce the required battery capability by 14.7 percent, compared to MAX, a naive approach without utilizing renewable energy sources.

### 7.2.2 Online Power Management

We target the case study explained in Section 7.1, and apply the simulation model to power demand and supply scheduling with our online management schemes. The worst-case power-demand operations are scheduled according to Table 2. The generated power is supplied into the energy buffer according to power-supply sources in Table 1. Fig. 8 shows the square-shaped worst-case power demand and supply results. We also generated more realistic total power demand and supply based on actual power supply/demand observations; we smoothed them and added uniform random noises. We then evaluated the schemes with ideal power supply and demand profiles and their smoothed profiles. That is, we consider two situations: (Sit1) the power demand and supply always exhibit their worst-case; and (Sit2) the power demand and supply include noises, which is more realistic. Under these situations, we have evaluated the following four online power management schemes:

8. Note that Plain–Ded – ii and Improved–Ded – ii choose \(\lambda_4\) to which the additional power-supply sources are dedicated.

- **STOM–Uni**: static-demand-based online management with the uniform supply,
- **STOM–Ded**: static-demand-based online management with the dedicated supply,
- **PDOM–Uni**: predicted-demand-based online management with the uniform supply, and
- **PDOM–Ded**: predicted-demand-based online management with dedicated supply.

We first evaluate the situation Sit1. Fig. 9 shows the result battery current under worst-case power supply and demand with the proposed online management methods, where \(P_{rq}\) and \(P_{bat}\) denote the total power demand and the amount of power supplied by the battery pack, respectively. STOM–Uni reduces battery power requirement evenly, while STOM–Ded decreases battery power requirement only when the dedicated tasks runs. PDOM–Uni and PDOM–Ded keep updating the current bound and limit battery current based on the current bound. STOM–Uni, STOM–Ded, PDOM–Uni, and PDOM–Ded reduce peak current to 3.44, 0.72, 7.30, and 6.01 percent, respectively, compared to \(P_{rq}\). Detailed values under Sit1 can be seen in Fig. 10a.

We also note the average power dissipation is calculated by:

\[
P_{loss} = \frac{1}{t_{max}} \int_0^{t_{max}} P_{bat-\text{loss}}(t) dt = \frac{1}{t_{max}} \int_0^{t_{max}} \sum R_{bat,i}(t) I_{bat,i}(t) dt.
\]

STOM–Uni, STOM–Ded, PDOM–Uni, and PDOM–Ded decrease the average power dissipation by 14.07, 14.23, 23.88 and 22.01 percent, respectively, compared to \(P_{rq}\). Detailed values under Sit1 can be seen in Fig. 10b.

Next, we evaluate the situation Sit2. Fig. 11 represents battery power of each approach with the more realistic power requirements. Fig. 12 shows the detailed peak power reductions. STOM–Uni, STOM–Ded, PDOM–Uni, and PDOM–Ded could decrease peak current respectively to 23.36, 22.44, 42.98 and 44.93 percent during the given operations, compared to \(P_{iq}\). The average power dissipation of STOM–Uni, STOM–Ded, PDOM–Uni, and PDOM–Ded are reduced to 32.60, 29.48, 55.79, and 56.49 percent, respectively, compared to \(P_{rq}\). Fig. 10a and (b) under Sit2 show the performance of the approaches under Sit2.
We can interpret the evaluation results as follows. First, if we compare the reduction of the peak current by the four online power management schemes in Sit1, with that in Sit2, that in Sit2 more significant than that in Sit1. This is because Sit2 exhibits less power consumption of power-demand operations than their worst-case and more power generation by power-supply sources than their minimum; this yields more room to reduce the peak current, and our proposed online management schemes effectively exploit the room. Second, if we compare STOM−Δ and PDOM−Δ, the latter yields better performance, since PDOM−Δ decides the current upper bound in real-time based on the average and standard deviation of recent current requirement instead of using worst-case task power requirement. This enables PDOM−Δ to reduce the peak current more aggressively while providing the buffer energy to the load without the buffer energy shortage. This improved peak power reduction leads to the lower average energy dissipation compared to STOM−Δ. Third, while −Δ−Uni reduces the current uniformly, so peak current is reduced by a certain amount, the peak current reduction of −Δ−Ded depends on which task is dedicated. Since the dedicated task \( \lambda_d \) (in the case study) did not contribute the peak current in Sit1 and Sit2, −Δ−Uni would be more beneficial for peak current reduction. However, if dedicated task contributes the system peak current, −Δ−Ded could reduce peak power largely. Results in Figs. 9 and 11 show this trend clearly.

## 8 Conclusion

In this paper, we have proposed a power scheduling framework—as a guideline for the design of a HESS—that ensures power sufficiency for the worst-case power demand and supply using real-time scheduling schemes. To improve the runtime performance of the HESS further, we have also designed an online power management framework that utilizes the surplus energy from a real-time power supply, reducing the battery’s peak power and hence extending its lifetime. We have evaluated the effectiveness of the offline power-supply guarantee using simulations, and validated the design with a simulation model based on the HESS-powered actual system running realistic applications, demonstrating power sufficiency with a lower-cost HESS and higher energy-efficiency.

In future, we would like to develop a design framework for general energy storage systems based on a power guarantee analysis. It will search for the optimal configurations of energy storage systems at their design or replacement time while considering power demand history and power supply’s state-of-health. We also plan to build a power/energy management system that not only schedules power demand and supply, but also monitors and pro/diagnoses energy storages/sources.

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## References


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