

Performance Evaluation of MISO-SDMA in Heterogenous Networks with Practical Cell Association

Mohammad G. Khoshkholgh*, Keivan Navaie[‡], Kang G. Shin[†], Victor C. M. Leung*

*The University of British Columbia (m.g.khoshkholgh@gmail.com, vleung@ece.ubc.ca)

[‡]Lancaster University (k.navaie@lancaster.ac.uk), [†]The University of Michigan (kgshin@umich.edu)

Invited Paper

Abstract—In this paper adopting stochastic geometry we investigate the system performance in heterogenous networks including multiple tiers of BSs with multiple-input single output spatial division multiple access (MISO-SDMA) technique. In the related literature on heterogenous systems, ideal cell association (CA) rules are often considered for simplicity, where each user equipment (UE) examines a very large number of pilots across the tiers before choosing its associated base station (BS). Here we consider practical cases where UEs are restricted to examine $K_H \geq 1$ pilots across all tiers before choosing their associated BS. We then obtain closed-form expressions for the system performance measured by the coverage probability and UE's data rate. Our analytical results provide quantitative insights on the impact of different factors on the system performance including the BS's spatial density, their transmission powers, number of transmit antennas, SIR thresholds, number of UEs served by each BS, and K_H . Interestingly, we observe that increasing K_H always improves the coverage probability however, it only improves data rate up to a certain point. The data rate is then reduced by further increasing of K_H . Given K_H pilots in practical cases, the issue is how to allocate the pilots among different tiers. We address this issue by developing an algorithm and show that by careful allocation of available pilots, the network performance is significantly improved even in cases with small K_H . Our results also indicate a fundamental tradeoff, as sharing strategies providing the best coverage performance yield very poor capacity and vice versa. Such trade-off provides a new degree of freedom in heterogeneous networks design.

I. INTRODUCTION

Cellular systems have been under radical design shifts in order to handle the occurring traffic demands. Current perspectives strongly advocate the installation of as many as possible small cells in the coverage area of macro cells in order for efficient traffic offloading [1]. Measurements suggest that about 60% of the traffic could conceivably be offloaded simply by installing a handful number of femto-cells over a macro cell. An immediate consequence is then the substantial growth of the networks spectral efficiency and/or energy efficiency. On the other hand, backed with extensive researches in two previous decades, more capacity growth is expecting to be realized by gearing up BSs with multiple antennas [2, 3], which has sparked broad researches in academia and industry.

The evaluation of the network's performance of MIMO communications in HetNets is an active research trend. Exploiting benefits of stochastic geometry, see, e.g., [4] for details

of the theory and several applications in wireless communications analysis, various aspects of MIMO techniques have been extensively investigated. Here we do not present all due to space limitations. Relevant literature to the particular subject of this paper—which is the evaluation of the network-wise performance of spatial division multiple access (SDMA) in HetNets—is then provided in the following. The focus in [5] was the single macro cell system overlaid by a number of multi-antenna femto cells. Work of [6] considered the design of uplink/downlink MIMO SDMA two-tier HetNets in order for the optimization of network's energy efficiency. Traits of wireless backhauling was also incorporated in the design. By assuming maximum signal-to-interference (SIR) cell association (CA) rule in [7] authors then provided some ordering results on the coverage probability of SDMA systems. The evaluations suggested that in many practical regimes SDMA is inferior to single-user beamforming. Fractional frequency reuse was also investigated in [8] in SDMA system.

However, in all the above mentioned work likewise many other relevant work in the field of stochastic geometry in cellular networks, see, e.g., [9–12], a very limiting assumption is made for modeling CA: in each communication frame that the network's status changes—due for instance to mobility, congestions, and fading—user equipments (UEs) are assumed stayed associated with the *best* BS, which is selected out of the entire pool of BSs. Ignoring congestion-driven CA/handoff, the BS providing the maximum average pilot power (range expansion method) [12–15] or the BS offering the maximum SIR [7, 8, 10] is regarded as the serving BS. But, to find this best BS countably infinite number of measurements of the emitted pilot signals from all the BSs is required, which is an unrealistic assumption and imposes staggering complexities and ultimately depletes the resources. In effect, the entire frame time must be designated for merely the CA mechanism, and unfortunately nothing will be left for actual data transmission—zero spectral efficiency.

In practice however, UEs are advocated to check the feasibility of *a number of adjacent BSs for association*. Accordingly, our goal in this paper is to investigate the impact of practical CA rule on the performance of SDMA in HetNets. To do this, we derive the coverage probability and per UE data rate taking into account the maximum number of BSs across

all tiers that UEs are allowed to assess for association, K_H . It is seen that, the coverage probability improves substantially by increasing K_H before reaching its stable status. But, by increasing K_H the data rate linearly diminishes to zero after reaching its summit. We further show that the way that K_H is shared among the tiers can dramatically improve the coverage and capacity. In fact, adopting our proposed algorithms one may achieve almost 90% of the stable coverage (which is obtainable when $K_H \gg 1$) only when K_H is 2. We further observe that methods enhancing coverage probability may render a very weak capacity performance and vice versa.

II. SYSTEM MODEL

Our main focus is on the downlink communication paradigm in a K -tier HetNet. As in [10, 11], a tractable network model of HetNets consists of K tiers of randomly located BSs. Each tier i is specified by tuple $(\lambda_i, P_i, \beta_i \geq 1, N_i^t, M_i)$, respectively, indicating BS's spatial density, transmission power of BSs, SIR threshold, the number of antennas BSs are equipped with, and the number of UEs served in each cell by SDMA method. BSs at the tier i are spatially distributed via a homogenous Poisson Point Process (PPP) $\Phi_i \in \mathbb{R}^2$ with given spatial density $\lambda_i \geq 0$. The processes are mutually independent. UEs are single-antenna and distributed through a homogenous PPP Φ_U , independent of sets Φ_i , with given spatial density λ_U .

We focus on narrow-band, block-fading model in which fading is constant per frame duration and evolves randomly according to the specified fading distribution at the start of each frame. The typical UE is positioned at the origin that is presumably associated with BS x_i . We here focus on zero-forcing precoding. Let h_{x_i} be the small-scale channel power gain between BS $x_i \in \Phi_i$ and the typical UE, which according to [7, 8] is a chi-squared *r.v.* with $2\hat{M}_i$ degrees-of-freedom (DoFs) where $\hat{M}_i = N_i^t - M_i + 1$. Other BSs all over the network are potential interferers under the premise of universal frequency reuse and open access. The experienced signal to interference ratio (SIR) at the typical UE is

$$\text{SIR}_{x_i} = \frac{\frac{P_i}{M_i} \|x_i\|^{-\alpha} h_{x_i}}{\sum_{j=1}^K I_j}, \quad (1)$$

where $\alpha > 2$ is the path-loss exponent, $\|x_i\|$ stands for the Euclidean distance. We further let $\check{\alpha} = \frac{2}{\alpha}$ in the rest of this paper. Also, $I_j = \sum_{x_j \in \Phi_j/x_0} \frac{P_j}{M_j} \|x_j\|^{-\alpha} g_{x_j}$ is the interference contribution of tier j . Fading g_{x_j} is a chi-squared *r.v.* with DoFs $2M_j$ [7]. Note that all involved fading variables in (1) are statistically independent.

By CA procedure UEs are able to examine pilot signals emitted from BSs and associate themselves with the most suitable one. As [7, 10] maximum SIR association is considered: each UE attaches to the BS that is able to provide the *highest* SIR. But, here we assume that the network is designed in the way that the assessment of solely $K_H \geq 1$ SIR values is permitted before finding the best one. This is inline with relevant issues dominating the practical scenarios, pilot's

reuse factor, and network's resource constraints. Further, let $0 \leq n_i \leq K_H$ be the number of i -th tier pilots that the typical UE measure so that $\sum_i n_i = K_H$. We introduce set Φ_i^H as the index of BSs of tier i that UE assesses for CA procedure. Regarding the fact that in practice pilots corresponding to *nearby* BSs have the higher chance of being successfully detected and network's configuration advocates handoff to adjacent cells rather far cells, set Φ_i^H is assumed to contain the n_i nearest BSs to the typical UE.

III. PERFORMANCE ANALYSIS

A. Coverage Probability

The typical UE is in coverage if for some i $\max_{x_i \in \Phi_i^H} \text{SIR}_{x_i} \geq \beta_i$ is valid. We denote the coverage probability by $o = \mathbb{P} \left\{ \max_{x_i \in \cup_i \Phi_i^H} \text{SIR}_{x_i} \geq \beta_i \right\}$.

Proposition 1: The coverage probability of the considered SDMA K -tier HetNets can be lower-bounded by

$$o \geq \sum_i \frac{\pi \tilde{\kappa}_i}{\tilde{C}(\alpha) \sum_{j=1}^K \kappa_j(M_j)} \sum_{m=0}^{\hat{M}_i-1} \left(\frac{\Gamma(\check{\alpha} + m)}{\Gamma(\check{\alpha})\Gamma(1+m)} \right. \\ \left. - \frac{(-1)^m}{m!} \frac{d^m}{ds^m} \frac{s^{-\check{\alpha}}}{\left(1 + \frac{\tilde{C}(\alpha)}{\pi s^{-\check{\alpha}} \tilde{\kappa}_i} \sum_{j=1}^K \kappa_j(M_j) \right)^{n_i}} \Big|_{s=1} \right), \quad (2)$$

where $\Gamma(\cdot)$ is the gamma function and $\tilde{\kappa}_i(M_i) = \lambda_i \left(\frac{P_i}{M_i} \right)^{\check{\alpha}} \beta_i^{-\check{\alpha}}$, $\kappa_j(M_j) = \lambda_j \left(\frac{P_j}{M_j} \right)^{\check{\alpha}} \frac{\Gamma(\check{\alpha} + M_j)}{\Gamma(M_j)}$, and $\tilde{C}(\alpha) = \pi \Gamma(1 - \check{\alpha})$.

Proof: See Appendix. \square

Proposition 1 provides an expression of the coverage probability that the typical UE experiences in SDMA systems. Impacts of various system parameters including the density of BSs, SIR thresholds, and specially n_i are apparent from (2). For example, by increasing n_i the significance of the second term in the summation reduces and the first term dominates the behavior of the coverage performance. As a result, by increasing n_i the coverage performance improves. However, under the constraint of $\sum_i = K_H$ it is in practice impossible to increase n_i s as freely as it wishes. It is in fact very critical to intelligently share K_H among the tiers according to the manners that tiers affect the coverage probability. For instance, one may decide to give a bigger portion of K_H to the tier responsible with smaller values of the second term in (2). Another approach could be sharing K_H among the tiers based on the levels of their transmission powers. We pursue these agendas in the design of the CA procedure with more details in Section IV.

Before ending this part we provide some corollaries to Proposition 1:

Corollary 1: If $n_i \gg 1 \forall i$, then (2) is increased to

$$o_{K_H=\infty} = \sum_{i=1}^K \frac{\pi \tilde{\kappa}_i(M_i)}{\tilde{C}(\alpha) \sum_{j=1}^K \kappa_j(M_j)} \sum_{m=0}^{\hat{M}_i-1} \frac{\Gamma(\check{\alpha} + m)}{\Gamma(\check{\alpha})\Gamma(1+m)}.$$

Corollary 2: For full SDMA, i.e., $M_i = N_i^t \forall i$, (2) is simplified to $o^{\text{F-SDMA}} \geq \sum_i A_i (1 - (1 + \frac{1}{A_i})^{-n_i})$, where $A_i =$

$$\frac{\pi \tilde{\kappa}_i}{\sum_{j=1}^K \kappa_j(M_j)}$$

According to Corollary 2 one may argue that $A_i(1 + \frac{1}{A_i})^{-n_i}$ is the damage that limited CA procedure imposes on the coverage performance of tier i . Since function $A_i(1 + \frac{1}{A_i})^{-n_i}$ is increasing with respect to A_i , one may decide to assign a bigger portion of K_H to the tier with smallest A_i across tiers.

B. Per-UE Capacity

The amount of time that the typical UE spends for association is proportional to the value of K_H . In the following we consider a very straightforward scenario. Let $0 < \tau \leq 1$ be the fraction of time frame that is devoted for measuring pilot from a BS. Further, let $\mathbf{C} = \{\max_{x_i \in \Phi_i^H} \text{SIR}_{x_i} \geq \beta_i\}$ stand for the coverage event of the typical UE. The capacity that the typical UE can achieve is then obtained from

$$\begin{aligned} R &= (1 - K_H \tau)^+ \mathbb{E} \left[\log \left(1 + \max_{x_i \in \Phi_i^H} \text{SIR}_{x_i} \right) \middle| \mathbf{C} \right] \\ &= (1 - K_H \tau)^+ \int_{t>0} \mathbb{P} \left\{ \max_{x_i \in \Phi_i^H} \text{SIR}_{x_i} > 2^t - 1 \middle| \mathbf{C} \right\} dt \\ &= \frac{(1 - K_H \tau)^+}{o} \int_{t>0} \mathbb{P} \left\{ \max_{x_i \in \Phi_i^H} \text{SIR}_{x_i} > \max\{2^t - 1, \beta_i\} \right\} dt \\ &= (1 - K_H \tau)^+ \log(1 + \beta_{\min}) + \frac{(1 - K_H \tau)^+}{o} \sum_i \int_{t>\log(1+\beta_{\min})} \sum_{l=1}^{n_i} \mathbb{P} \left\{ \text{SIR}_{x_i^{(l)}} \geq \max\{2^t - 1, \beta_i\} \right\} dt, \quad (3) \end{aligned}$$

where $\beta_{\min} = \min_i \beta_i$ and $\mathbb{P} \left\{ \text{SIR}_{x_i^{(l)}} \geq \max\{2^t - 1, \beta_i\} \right\}$ is calculated in Proposition 1.

IV. SIMULATION RESULTS AND SYSTEM DESIGN

The way that K_H is shared among tiers has a profound impact on the coverage probability as well as per-UE capacity. While optimizing (2) and/or (3) with respect to n_i s is mathematically complex, we in the following provide a greedy algorithm that can be found in Algorithm 1 (referred to as Alg. 1 in the following). In Alg.1 parameter ψ_i shall be interpreted as the performance reward gained by assigning a new BS for CA procedure to tier i . Possible choices are: $\psi_i = o_i$ (the coverage probability associated with tier i); $\psi_i = R_i$ (the per-UE capacity associated with tier i); and $\psi_i = A_i^{-1}$ where A_i is given in Corollary 2. We also consider the case that $\psi_i = P_i$ meaning that a tier with higher transmission power has the higher superiority to have higher share of K_H .

In the rest of this section we evaluate the performance of Alg. 1 for several choices of ψ_i . However, we first study the accuracy of Proposition 1's bound against simulations¹. For

¹For the simulations in this paper we have considered the following parameters: $\lambda_1 = 10^{-5}$, $\lambda_2 = 10^{-4}$, $\lambda_3 = 5 \times 10^{-4}$, $\lambda_4 = 5 \times 10^{-3}$, $P_1 = 50$, $P_2 = 10$, $P_3 = 5$, $P_4 = 1$, $\beta_1 = 2.5$, $\beta_2 = 5$, $\beta_3 = 5$, $M_1 = 3$, $M_2 = 5$, $M_3 = 7$, $M_4 = 2$, and $N_i^t = 8 \forall i$.

Algorithm 1 Alg. 1

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1:  $t = 0, \hat{\mathcal{K}} = \emptyset$ 
2: while  $\sum_i n_i \leq K_H$  do
3:    $i^* = \arg \max_{i \in \mathcal{K}/\hat{\mathcal{K}}} \psi_i$ 
4:    $n_{i^*} = n_{i^*} + 1$ 
5:    $\hat{\mathcal{K}} = \hat{\mathcal{K}} \cup \{i^*\}$ 
6:    $t = t + 1$ 
7:   if  $\hat{\mathcal{K}} = \mathcal{K}$  then
8:      $\hat{\mathcal{K}} = \emptyset$ 
9:   end if
10: end while

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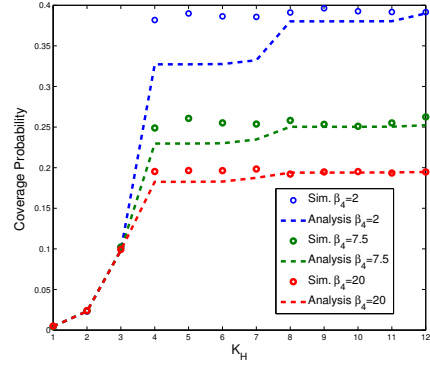


Fig. 1. Coverage Probability vs. K_H for several values of β_4 (Alg1. with $\psi_i = P_i$).

this goal we simply let $\psi_i = P_i$ in Fig. 1. As it is seen from this figure the provided lower-bound in Proposition 1 is adequately accurate. Moreover, it is seen that by increasing K_H the coverage probability improves and then gets stable. In fact, as many as 8 BSs are enough for CA procedure from the coverage perspective.

A plot of per-UE capacity of (3) for several values of β_4 is shown in Fig. 2 vs. K_H under the algorithm Alg. 1. As it is seen, in contrary to coverage performance (see Fig. 1) increasing K_H has a conflicting impact on R : it first improves the capacity and then proportionally pushes its value to zero. In the case of $\beta_4 = 2$ setting $K_H = 2$ in fact results in the maximization of the capacity. Nevertheless, Fig. 1 reveals that for this specific setting the coverage performance is very poor.

We further study the performance of Alg. 1 for several choices of parameter ψ_i in Fig. 3, which shows the coverage performance, and Fig. 4, which shows the per-UE capacity.

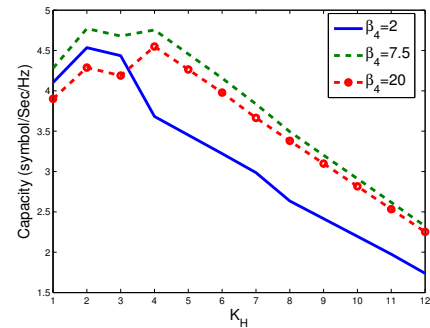


Fig. 2. Per-UE capacity vs. K_H for several values of β_4 (Alg1. with $\psi_i = P_i$).

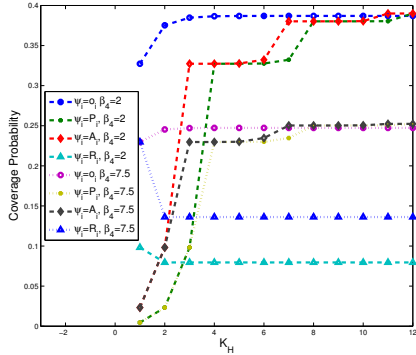


Fig. 3. $\lambda_1 = 10^{-4}$. $\alpha = 4$, $N^r = 10$, $P_1 = 50$ W, $P_2 = 10$ W, $\beta_1 = 2$, and $\beta_2 = 5$.

Consider Fig. 3. As it is seen, when $\psi_i = o_i$ the coverage performance is the highest compared to the other choices for each value of β_4 . For example, when $\beta_4 = 2$, it is observable that only 3 BSs are required for CA procedure to guarantee almost the same coverage that $K_H \rightarrow \infty$ can provide. This is also true for the case of $\beta_4 = 7.5$. On the other hand, for both values of β_4 we see that choice of $\psi_i = R_i$ performs very poorly. Finally, if $\psi_i = 1/A_i$, the coverage performance improves compared to $\psi_i = P_i$. For both choices of $\psi_i = 1/A_i$ and $\psi_i = P_i$ and in both cases of $\beta_4 = 2$ and $\beta_4 = 7.5$ we require $K_H = 8$ to achieve ultimate coverage performance. As a result, a wise choice of parameter ψ_i can dramatically reduce the complexity of CA procedure without damaging the coverage performance.

Now consider Fig. 4 that depicts the capacity performance of Alg. 1 for different choices of ψ_i . There are a number of important points deserved to be explained. First, as a general rule the larger the value of K_H , the smaller the capacity will be. As a result, though the coverage performance benefits from higher values of K_H the same can not be claimed from the perspective of capacity. Second, the choices of ψ_i that performs great from the capacity point of view, which is $\psi_i = R_i$, yields a very poor coverage performance (see Fig. 3), while choice of $\psi_i = o_i$ that has the best coverage performance yields the weakest capacity performance (see Fig. 4). In fact, from capacity view point, see Fig. 4 for $\beta_4 = 7.5$, the capacity boost one may gain from changing the parameter ψ_i from o_i to R_i is more than 3 nats/sec/Hz, while the coverage performance degrades more than 15%. Third, choices of $\psi_i = P_i$ and $\psi_i = 1/A_i$ have almost the same capacity performance. Besides, their corresponding performance is somewhere between capacity of $\psi_i = R_i$ and capacity of $\psi_i = o_i$. It is important to compare Fig. 3 with Fig. 4 to realize that these two choices yields their best capacity performance—for example $K_H = 3$ for the case of $\beta_4 = 2$ —with the cost of less than 10% coverage lost (compared to the coverage that choice $\psi_i = o_i$ yields). On the other hand, for this case the capacity lost compared to $\psi_i = R_i$ ($K_H = 3$ and $\beta_4 = 2$) is less than 1 nats/sec/Hz. As a result, the conclusion one may draw here is that to balance between two conflicting

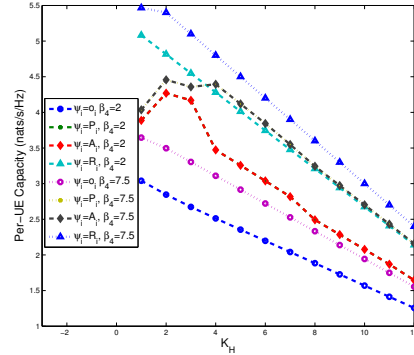


Fig. 4. $\lambda_1 = 10^{-4}$. $\alpha = 4$, $N^r = 10$, $P_1 = 50$ W, $P_2 = 10$ W, $\beta_1 = 2$, and $\beta_2 = 5$.

performance metrics of coverage and capacity it is better to have a moderate number of BSs for CA procedure, which could be chosen with accordance of $\psi_i = 1/A_i$.

V. CONCLUSIONS

Adopting tools from stochastic geometry we derived the coverage probability and capacity of MIMO-SDMA in Het-Nets. We mainly focused on practically appealing scenarios whereby user equipments (UE) were restricted examining $K_H \geq 1$ pilots across all tiers before choosing the associated base station (BS) in contrary to the literature that have considered idealistic cell association (CA) rules, i.e., $K_H \rightarrow \infty$. Adopting our proposed bounds on the coverage probability and capacity we then concocted a number of greedy algorithms for sharing K_H among the tiers.

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APPENDIX: PROOF OF PROPOSITION 1

Let sort the BSs in tier i such that $\|x_i^{(l)}\| \leq \|x_i^{(l+1)}\|$ where $x_i^{(l)}$ is the position of l -th closest BS to the origin. Note that *r.v.* $\pi\lambda_i\|x_i^{(l)}\|^2$ is distributed according to $2l$ chi-squared [16]. Under the premise of $\beta_i \geq 1$, the coverage probability is then calculated as

$$\begin{aligned} o &= \sum_{i=1}^K \mathbb{E} \left[\sum_{x_i \in \Phi_i^H} 1(\text{SIR}_{x_i} \geq \beta_i) \middle| \{\Phi_j^H\} \right] \\ &\geq \sum_{i=1}^K \mathbb{E} \left[\sum_{x_i \in \Phi_i^H} 1(\text{SIR}_{x_i} \geq \beta_i) \middle| \Phi_i^H \right] = \sum_{i=1}^K \sum_{l=1}^{n_i} \mathbb{P} \left\{ \text{SIR}_{x_i^{(l)}} \geq \beta_i \right\} \end{aligned}$$

in which we have applied the whole probability formula and the fact that at most n_i candidate BSs in tier i are assessable for CA procedure. We then have

$$\begin{aligned} \mathbb{P} \left\{ \text{SIR}_{x_i^{(l)}} \geq \beta_i \right\} &= \int_0^\infty f_{\chi_{2l}^2}(x) \mathbb{P} \left\{ h_{x_i} \geq \frac{\beta_i x^{\alpha/2}}{(\pi\lambda_i)^{\frac{\alpha}{2}} \frac{P_i}{M_i}} \sum_{j=1}^K I_j \right\} dx \\ &= \int_0^\infty f_{\chi_{2l}^2}(x) \mathbb{E}_{\{I_j\}} \int_0^\infty \mathcal{L}_{\bar{F}_{\chi_{2l}^2}^2}^{-1}(s) e^{-\frac{\beta_i x^{\alpha/2}}{(\pi\lambda_i)^{\frac{\alpha}{2}} \frac{P_i}{M_i}} \sum_{j=1}^K I_j} ds dx, \quad (4) \end{aligned}$$

where $\mathcal{L}_{\bar{F}_{\chi_{2l}^2}^2}^{-1}(s)$ is the Laplace transform of CCDF of random

variable χ_{2l}^2 $\mathcal{L}_{\bar{F}_{\chi_{2l}^2}^2}^{-1}(s) = \sum_{m=0}^{\hat{M}_i-1} \frac{1}{m!} \delta^{(m)}(s-1)$. such that

$\int_0^\infty e^{-sh} \mathcal{L}_{\bar{F}_{\chi_{2l}^2}^2}^{-1}(s) ds = e^{-h} \sum_{l=0}^{\hat{M}_i-1} \frac{h^l}{l!}$, and $\delta^{(m)}(t)$ is the m -th derivative of Dirac delta function [17]. Then,

$$\begin{aligned} (4) &= \int_0^\infty f_{\chi_{2l}^2}(x) \int_0^\infty \mathcal{L}_{\bar{F}_{\chi_{2l}^2}^2}^{-1}(s) \prod_{j=1}^K \mathcal{L}_{I_j} \left(s \frac{\beta_i x^{\alpha/2}}{(\pi\lambda_i)^{\frac{\alpha}{2}} \frac{P_i}{M_i}} \right) ds dx \\ &\stackrel{(b)}{=} \int_0^\infty f_{\chi_{2l}^2}(x) \int_0^\infty \mathcal{L}_{\bar{F}_{\chi_{2l}^2}^2}^{-1}(s) e^{-\left(s \frac{\beta_i}{P_i} \frac{P_i}{M_i}\right)^{\frac{\alpha}{2}} \frac{\tilde{C}(\alpha)}{\pi\lambda_i} \sum_{j=1}^K \kappa_j(M_j) x} ds dx \end{aligned}$$

$$\stackrel{(c)}{=} \int_0^\infty \mathcal{L}_{\bar{F}_{\chi_{2l}^2}^2}^{-1}(s) \left(1 + \left(s \frac{M_i \beta_i}{P_i} \right)^{\frac{\alpha}{2}} \frac{\tilde{C}(\alpha)}{\pi\lambda_i} \sum_{j=1}^K \kappa_j(M_j) \right)^{-l} ds,$$

where in step (b) we introduce $\kappa_j(M_j) = \lambda_j P_j^{\frac{\alpha}{2}} \mathbb{E}[(\chi_{2M_j}^2)^{\frac{\alpha}{2}}]$ where $\mathbb{E}[(\chi_{2M_j}^2)^{\frac{\alpha}{2}}] = \frac{\Gamma(\frac{\alpha}{2} + M_j)}{\Gamma(M_j)}$, and plug the Laplace transform of random variable I_j that is $\mathcal{L}_{I_j}(t) = e^{-t^{\frac{\alpha}{2}} \tilde{C}(\alpha) \lambda_j \left(\frac{P_j}{M_j}\right)^{\frac{\alpha}{2}} \mathbb{E}[(\chi_{2M_j}^2)^{\frac{\alpha}{2}]}$, for $t = s \frac{M_i \beta_i \|x_i^{(l)}\|^{\frac{\alpha}{2}}}{P_i}$. Step (c) finally follows by some straightforward manipulations. As a result, $\sum_{l=1}^{n_i} \mathbb{P} \left\{ \gamma(x_i^{(l)}) \geq \beta_i \right\}$ is

$$\begin{aligned} &= \int_0^\infty \mathcal{L}_{\bar{F}_{\chi_{2l}^2}^2}^{-1}(s) \sum_{l=1}^{n_i} \left(1 + \left(s \frac{M_i \beta_i}{P_i} \right)^{\frac{\alpha}{2}} \frac{\tilde{C}(\alpha)}{\pi\lambda_i} \sum_{j=1}^K \kappa_j(M_j) \right)^{-l} ds \\ &= \int_0^\infty \mathcal{L}_{\bar{F}_{\chi_{2l}^2}^2}^{-1}(s) \frac{\pi s^{-\frac{\alpha}{2}} \tilde{\kappa}_i(M_i)}{\tilde{C}(\alpha) \sum_{j=1}^K \kappa_j(M_j)} \\ &\quad \times \left(1 - \left(1 + \frac{\tilde{C}(\alpha)}{\pi s^{-\frac{\alpha}{2}} \tilde{\kappa}_i(M_i)} \sum_{j=1}^K \kappa_j(M_j) \right)^{-n_i} \right) ds \\ &= \frac{\pi \tilde{\kappa}_i(M_i)}{\tilde{C}(\alpha) \sum_{j=1}^K \kappa_j(M_j)} \int_0^\infty \mathcal{L}_{\bar{F}_{\chi_{2l}^2}^2}^{-1}(s) s^{-\frac{\alpha}{2}} ds \\ &\quad - \frac{\pi \tilde{\kappa}_i(M_i)}{\tilde{C}(\alpha) \sum_{j=1}^K \kappa_j(M_j)} \int_0^\infty \frac{\mathcal{L}_{\bar{F}_{\chi_{2l}^2}^2}^{-1}(s) s^{-\frac{\alpha}{2}}}{\left(1 + \frac{\tilde{C}(\alpha)}{\pi s^{-\frac{\alpha}{2}} \tilde{\kappa}_i(M_i)} \sum_{j=1}^K \kappa_j(M_j) \right)^{n_i}} ds. \quad (5) \end{aligned}$$

Following the same procedure developed in [17] we can show that

$$\int_0^\infty \mathcal{L}_{\bar{F}_{\chi_{2l}^2}^2}^{-1}(s) s^{-\frac{\alpha}{2}} ds = \sum_{m=0}^{\hat{M}_i-1} \frac{\Gamma(\frac{\alpha}{2} + m)}{\Gamma(\frac{\alpha}{2}) \Gamma(1 + m)}. \quad (6)$$

On the other hand,

$$\begin{aligned} &\int_0^\infty \frac{\mathcal{L}_{\bar{F}_{\chi_{2l}^2}^2}^{-1}(s) s^{-\frac{\alpha}{2}}}{\left(1 + \frac{\tilde{C}(\alpha)}{\pi s^{-\frac{\alpha}{2}} \tilde{\kappa}_i(M_i)} \sum_{j=1}^K \kappa_j(M_j) \right)^{n_i}} ds \\ &= \sum_{m=0}^{\hat{M}_i-1} \frac{1}{m!} \int_0^\infty \frac{\delta^{(m)}(s-1) s^{-\frac{\alpha}{2}}}{\left(1 + \frac{\tilde{C}(\alpha)}{\pi s^{-\frac{\alpha}{2}} \tilde{\kappa}_i(M_i)} \sum_{j=1}^K \kappa_j(M_j) \right)^{n_i}} ds \\ &= \sum_{m=0}^{\hat{M}_i-1} \frac{(-1)^m}{m!} \frac{d^m}{ds^m} \left. \frac{s^{-\frac{\alpha}{2}}}{\left(1 + \frac{\tilde{C}(\alpha)}{\pi s^{-\frac{\alpha}{2}} \tilde{\kappa}_i(M_i)} \sum_{j=1}^K \kappa_j(M_j) \right)^{n_i}} \right|_{s=1}. \quad (7) \end{aligned}$$

By combining (7), (6), and (5) the desired result will be obtained.