# On the Impact of Delay Constraint on the Multicast Outage in Wireless Fading Environment

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***Abstract*—In this paper we investigate single-hop multicast transmission in which randomly located multiple transmitters multicast packets to a cluster of receivers. Packet retransmission is known as a promising mechanism for improving the transmission reliability. Our focus is on evaluating (i) the minimum required delay (retransmission attempts), $\tau^*$, for establishing an outage-free multicast, where a transmitted packet is successfully decoded by entire nodes in the cluster, and (ii) Multicast Progress Radius (MPR) for a given delay constraint. MPR indicates how far, on average, a packet can successfully progress in a cluster without outage while the retransmission delay is restricted. Assuming general fading distribution, we derive closed-form expressions for the cumulative distribution function of $\tau^*$, and MPR. By simulations we confirmed our analysis and studied the impact of several system parameters on the MPR. Based on results of this paper we conclude that outage-free multicast requires a very large number of retransmission attempts, thus not practically achievable only based on retransmission.**

## I. Introduction

Multicast-enabled technologies such as IPTV, mobile TV, and video conferencing provide bandwidth-efficient scheduling and resource allocation for group-based data communications and multimedia applications [1]. Services including geographic information updates, such as weather forecasting and traffic reports also utilize multicast communications [2]. In Machine-to-Machine (M2M) and smart grid communications, energy tariffs and bills should be multicasted to many destination nodes to facilitate optimal decision making and environmentally-friendly usage of electric energy [3].

Understanding multicast transmission and its corresponding fundamental limits, such as multicast throughput/capacity, devising efficient scheduling techniques and access policies are challenging tasks, see, e.g., [1], [4]. Multicasting implies that the same transmitted packet has to be received by all the destination receivers, in which re-transmissions might be needed.

Queuing characteristics and capacity bounds of multicast networks are among investigated issues in this research area, see, e.g., [5], [6], [7]. The maximization of throughput under system stability and packet loss is also investigated in [5] and a threshold-based policy is proposed for ready receivers. For the case of memoryless erasure channels, the authors of [6] analyze the delay performance of a block-based random linear combination of packets. Bounds on multicast capacity are derived in [4], [8], [9] and the scaling laws are then provided to study the asymptotic behavior of the multicast throughput when the number of nodes grows unboundedly. Moreover, the authors of [10] proposed bounds for the minimum energy required for reliably sending a multicast bit when bandwidth and delay are immaterial.

The techniques of analyzing multicast throughput/capacity in [4], [8], [9], [10] are the extensions of those in [11] that is used to analyze the bound on unicast *transport capacity* in a wireless ad hoc network. This parameter is a well understood performance metric, particularly in wireless ad hoc networks, and captures the aggregated achievable data rate weighted with the communication distance between transmitters and receivers. Nevertheless, outage probability and link failure – which multicasting is prone to – are almost overlooked. On the other hand, the *transmission capacity* suggested in [12], is designed to incorporate the impacts of outage events and link failures, and actually measures the area spectral efficiency [13].

For the multicast scenario, the authors of [14] derived the transmission capacity, which is defined as the maximum number of multicast sessions per unit area the network can accommodate subject to the multicast outage probability (MOP) and decoding delay (number of retransmission attempts) constraints. Tools from the cluster point processes [15] are used for modeling the multicast networks and deriving the MOP. The path-loss attenuation is then shown to be the main source of outage degradation that could be mitigated if the clusters are judiciously tessellated for local retransmissions of packets. Furthermore, the advantages of retransmission for boosting multicast transmission capacity are emphasized.

Nevertheless, in the previous studies a number of essential issues have unfortunately left behind. In essence, it is not obvious (i) how many retransmissions are exactly necessary before achieving outage-free multicast cluster; (ii) how effectively the outage-free zone in a cluster is affected by the retransmission attempts.

### A. Main Results

In this paper we aim to provide answers to the two above fundamental questions. We borrow the Poisson Cluster Process (PCP) introduced in [14] for modeling the multicast network. However, instead of evaluating multicast transmission capacity, in this paper we examine *outage-free multicast* and *multicast progress radius* (MPR). MPR indicates the radius of a cluster with vanishing multicast outage for given decoding delay.

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The main contributions of this paper are (i) evaluating the cumulative distribution function of the minimum required number of retransmissions for establishing an outage-free multicast, i.e., all the receiver nodes in a cluster are able to successfully decode the multicast packet; (ii) obtaining a closed-form expression for MPR for a given decoding delay.

In particular, the analytical results provided in this paper suggest that although outage-free multicast is a desirable, it may compromise the effective throughput of the network as the number of retransmission attempts required is rather high. Furthermore, the MPR increases quite slowly with increasing number of retransmission attempts. Consequently, one of the main conclusions that can be drawn based on the results in this paper is that a simple retransmission strategy is most likely not enough for achieving outage free multicast within a reasonable number of retransmissions. Therefore, more sophisticated scheduling strategies and network optimization and multi level error correcting mechanisms might be needed.

## II. Model and Definitions

In this paper we adopt the system model in [14]. Here however we utilize this model for investigating outage-free multicast communications.

The considered multicast communication scenario comprises a number of source nodes indexed by $i$, $X_i$. Each source nodes transmits packets to a set of destination nodes which is referred to as a *cluster*. Associated with a source node $i$, we define a disk $\mathcal{R}_i$ with radius $s > 1$ (in meters). Destination nodes are randomly placed in each cluster $i$ according to homogeneous marked Poisson Point Process (PPP), $\Phi_i^r$, with intensity measure $\lambda_r$. For any two clusters $i$ and $i'$, $\Phi_i^r$ and $\Phi_{i'}^r$ are independent PPPs. This model allows clusters to overlap, thus some clusters may contain other active sources and unintended destination nodes. In this model, spatial distribution of the source nodes also forms a homogeneous marked PPP, $\Phi_t$, with spatial density $\lambda_t$.

Corresponding to $X_i$,

$$\mathcal{Z} = \bigcup_{X_i \in \Phi_t} (\Phi_i^r \cup X_i),$$

is defined which forms a Poisson Cluster Process (PCP) with density $k\lambda_t$, where $k = \pi s^2 \lambda_r$ is the average number of receivers. For brevity here we focus on cluster $\mathcal{R}_0$ associated with source node $X_0$ located at the origin. Due to stationarity of PCP, the results can be easily extended to any source node in the network coverage area.

Set $\Phi_0^r = \{(Y_j, H_j), Y_j \in \mathcal{R}_0, H_j \geq 0, j \in \mathbb{N}_+\}$ is a collection of 2-tuples each include the destination node, $Y_j$, and a corresponding fading mark, $H_j$, representing the wireless channel power gain between $X_0$ and $Y_j$. Fading is also assumed to be location-independent with probability density function (pdf) of $f_H(.)$. Furthermore, transmitters form $\Phi_t = \{(X_i, \tilde{H}_{ij}), X_i \in \mathbb{R}^2, \tilde{H}_{ij} \geq 0, i \in \mathbb{N}_+, j \in \mathbb{N}_+\}$, where $\tilde{H}_{ij}$ is independent of $H_j$ and is equal to the interfering channel power gain between transmitter $i$ and receiver $j$ at the cluster $\mathcal{R}_0$ drawn from the same pdf, $f_H(.)$.

The quality of link $j \in \Phi_0^r$ is determined by Signal-to-Interference Ratio (SIR)–assuming all transmitters have the same transmission power and focusing on the interference-limited scenario–defined as

$$\mathrm{SIR}_j = \frac{H_j \|Y_j\|^{-\alpha}}{I_j}. \tag{1}$$

In (1), $\|Y_j\|$ is the Euclidian distance between transmitter $X_0$ and receiver $X_j$; $\alpha > 2$ is the path-loss exponent; $\|Y_j\|^{-\alpha}$ is the distance-dependent path-loss attenuation, and $I_j$ is the interference experienced at receiver $Y_j$ from all other active transmitters in the network. We note that this path-loss model is valid for $\|Y_j\| \leq 1$, however similar to [16], [17] we also use it for $\|Y_j\| < 1$ as it has a negligible effect in our derivations.

In this model time is slotted into frames and the sources adopt slotted ALOHA protocol for multi-access, where each source node independently decides whether or not to transmit. Interference, $I_j$, is therefore

$$I_j = \sum_{i \in \Phi_t / X_0} \tilde{H}_{ij} \|X_i - Y_j\|^{-\alpha}. \tag{2}$$

Outage is experienced if $\mathrm{SIR}_j < \beta$, where $\beta$ is the receiver SIR threshold. Following the same line of argument as in [14], one can show that it is sufficient to focus on the statistic of $I_0$–the aggregated interference measured at the origin.

Since the packets must be successfully decoded by all the destination nodes, in some cases the packet needs to be retransmitted. Maximum number of retransmissions is referred to *permissible decoding delay* $\tau$. Assuming unit time slot duration, $\tau$ also represents the maximum number of retransmissions.

## III. Multicast Outage

Multicast outage is experienced where the transmitted packet is not being successfully decoded by any of the destination nodes in the corresponding cluster. A key performance metric for evaluating the efficiency of the multicast communications is the *probability of multicast outage*.

Let *destination-connected set*, $\Omega_0$, be the collection of all destination nodes managed to successfully decode the packet at least once during $\tau$ transmission attempts. Definition of the destination-connected set is inspired by the similar concept of the receiver-connected process originally proposed in [14]. Random variable $\delta_j$ is associated to destination node $Y_j$ as

$$\delta_j = \sum_{t=1}^{\tau} \mathbf{1}\left(\mathrm{SIR}_j \geq \beta\right), \tag{3}$$

where $\mathbf{1}(.)$ is an indicator function. Note that $\delta_j$ is the number of occasions, where user $j$ is able to successfully decode multicast packets. Multicast outage occurs if for some $j \in \Phi_0^r$, $\delta_j = 0$. Therefore,

$$\Omega_0 = \{j \in \Phi_0^r : \delta_j > 0\}. \tag{4}$$

### A. Effect of the Number of Retransmissions

Due to independent fading and negligible spatial correlation in the received interferences, in each transmission attempt, destination nodes in a cluster are independently marked by $\mathbb{P}\{\text{SIR} \geq \beta\}$. Therefore, for a destination node located at $Y_j$ in $\Omega_0$, it is independently marked by $\mathbb{P}\{\delta_j > 0\}$, where

$$\begin{aligned}\mathbb{P}\{\delta_j > 0\} &= 1 - \mathbb{P}\left\{\sum_{t=1}^{\tau} \mathbf{1}\left(\text{SIR}_j(t) \geq \beta\right) = 0\right\} \\ &= 1 - \mathbb{P}\left\{\text{SIR}_j(t) < \beta, \forall t = \{1, \ldots, \tau\}\right\} \\ &= 1 - \prod_{t=1}^{\tau} \mathbb{P}\left\{\text{SIR}_j(t) < \beta\right\} \\ &= 1 - \left(\mathbb{P}\left\{\text{SIR}_j(t) \leq \beta\right\}\right)^{\tau} \\ &\overset{(a)}{=} 1 - \left(\mathbb{E}_I[F_H(\beta \|Y_j\|^{\alpha} I_0)]\right)^{\tau}. \end{aligned} \quad (5)$$

In (5), $(a)$ follows from the result of Eq. (36) in [14]. Hence the density of the points in $\Omega_0$ at location $\|Y_j\|$ is simply the multiplication of $\lambda_r$ and (5).

The multicast outage probability is equal to the void probability of the Poisson process:

$$\mathbb{P}\{\Omega_0 = \emptyset\} = \exp\left(-2\pi\lambda_r \int_0^s \mathbb{P}\{\delta_r > 0\} r dr\right). \quad (6)$$

Substituting (5) in (6), following with straightforward mathematical derivations, one can show:

$$\mathbb{P}\{\Omega_0 = \emptyset\} = \exp\left(-k[1 - \mathbb{E}_R\left(\mathbb{E}_I[F_H(\beta R^{\alpha} I_0)]\right)^{\tau}]\right), \quad (7)$$

where $R$ is a random variable with the support interval $[0, s]$ and pdf of $f_R(r) = 2r/s^2$.

As it is seen in (7) by increasing number of retransmissions, the probability of outage is also reduced. The question however is "how many retransmissions are required to have an outage-free multicast?"

The formation of the outage-free multicast coincides with the event by which all the receivers in a typical cluster happens to be able to successfully decode the multicast packet within $\tau$ retransmissions.

Let $\tau^*$ be a random variable denoting the minimum number of required retransmissions for attaining the outage-free multicast and $\Omega_0(t)$ be the destination-connected set for $t$ retransmissions, then

$$\begin{aligned}\mathbb{P}\{\tau^* = \tau\} &= e^{-k} \sum_{n=1}^{\infty} \frac{k^n}{n!} \mathbb{P}\Big\{ N[\Omega_0(\tau)] = n \\ &\qquad , N[\Omega_0(\tau - t)] < n, \forall t \in [1, \tau - 1] \Big\}, \end{aligned} \quad (8)$$

where $N[A]$ is cardinality of set $A$. Note that $\tau = \tau^*$, if at $\tau$ all the destination nodes in the cluster decode the packet successfully for the first time.

For a given realization of $\Phi_0^r$, sets $\Omega_0(t)$ are independent for different values of $t$ due to the independence of fading fluctuations and interference. Therefore,

$$\begin{aligned}\mathbb{P}\{\tau^* = \tau\} &= e^{-k} \sum_{n=1}^{\infty} \frac{k^n}{n!} \mathbb{P}\{N[\Omega_0(\tau)] = n\} \\ &\quad \prod_{t=1}^{\tau-1} \mathbb{P}\{N[\Omega_0(\tau - t)] < n\}. \end{aligned} \quad (9)$$

Using (5), (7),

$$\begin{aligned}\mathbb{P}\{N[\Omega_0(t)] = m\} &= e^{-2\pi\lambda_r \int_0^s \mathbb{P}\{\delta_r(t) > 0\} r dr} \\ &\quad \times \frac{(2\pi\lambda_r \int_0^s \mathbb{P}\{\delta_r(t) > 0\} r dr)^m}{m!}, \end{aligned} \quad (10)$$

where

$$\mathbb{P}\{\delta_r(t) > 0\} = 1 - \left(\mathbb{E}_I[F_H(\beta r^{\alpha} I_0)]\right)^t.$$

The Cumulative Distribution Function of random variable $\tau^*$ is then obtained using (10) in (9), which is valid for any fading distribution.

### B. Multicast Outage in Rayleigh Fading

For particular case of Rayleigh fading we can write

$$\mathbb{P}\{\delta_r(t) > 0\} = 1 - e^{-\pi\lambda_t \beta^{\frac{2}{\alpha}} \tilde{K}(\alpha) t r^2},$$

where $\tilde{K}(\alpha) = \Gamma(1 + 2/\alpha)\Gamma(1 - 2/\alpha)$. Therefore,

$$\mathbb{P}\{N[\Omega_0(t)] = m\} = \frac{1}{m!} e^{-k + \frac{\lambda_r e^{-\pi\lambda_t \beta^{\frac{2}{\alpha}} \tilde{K}(\alpha) t s^2}}{\lambda_t \beta^{\frac{2}{\alpha}} \tilde{K}(\alpha) t}}$$

$$\times \left(k - \frac{\lambda_r e^{-\pi\lambda_t \beta^{\frac{2}{\alpha}} \tilde{K}(\alpha) t s^2}}{\lambda_t \beta^{\frac{2}{\alpha}} \tilde{K}(\alpha) t}\right)^m.$$

Using this alongwith (9), a closed-form for $\mathbb{P}\{\tau^* = \tau\}$ is then obtained.

For other fading distributions, it might be hard to derive a closed-form expression. Later in this paper we have proposed a procedure to derive an approximation for $\mathbb{P}\{\delta_r(t) > 0\}$, which provides an approximation of $\mathbb{P}\{\tau^* = \tau\}$.

Looking at $P\{\tau^* \leq \tau\}$ in Fig. 1 it is seen, as expected, that retransmission in fact contributes toward reducing multicast outage probability. Surprisingly however, for larger number of re-transmissions, $\tau > 10$, the rate of increase in the probability of outage free multicast is rather low.

Based on the above we draw the conclusion that achieving outage-free multicast only based on re-transmissions results in very long delays due to very large number of required retransmissions. This significantly compromise multi-cast throughput performance. In practice however, only a limited number of retransmissions could be assumed feasible hence the notion of *partially outage-free multicast* is considered as a practical objective in designing multicast systems.

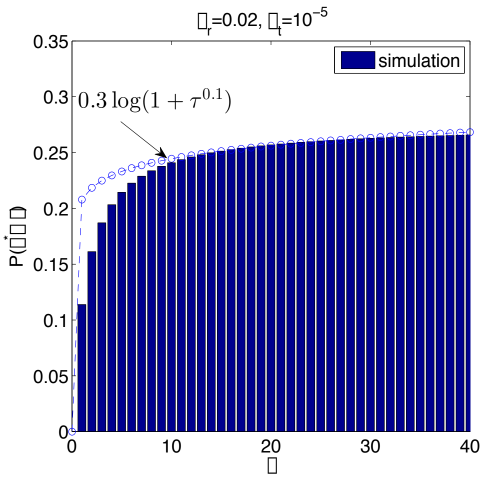


Fig. 1. $\mathbb{P}\{\tau^* \le \tau\}$ for the cluster radius $s = 10$.

## IV. Partially Outage Free Multicast

The performance of partially outage-free multicast, for a maximum value of $\tau$, is then directly related to the percentage of the destination nodes in a cluster that successfully decode the transmitted packet. To analyze this, here is this paper we define *multicast progress radius* (MPR) as a new performance metric:

***Definition** 1:* For a given maximum number of retransmissions, Multicast Progress Radius (MPR), $\eta$ is defined as the maximum average distance a multicast packet can successfully traverse in each cluster:

$$\eta \triangleq \mathbb{E}\left[\max_{j\in\Omega_0} \|Y_j\|\right]. \quad (11)$$

***Remark:*** Please note that MPR is reminiscent of the notion of *progress* for measuring single-hop unicast progress distance toward the eventual destination in multi-hop ad-hoc communication networks [16]. Apparently, the fundamental differences exist between the definitions and implications of MPR and progress, while progress may not be a meaningful metric for evaluating the performance of multicast in our system mdel. Note that MPR is not a trivial extension of progress introduced in [16] due to the following two reasons. Firstly, the multicast packet should be received by several destination nodes in a cluster so that the effective distance involves in the definition is actually the maximum distances between the transmitter and outage-free receivers. Secondly, since a packet is retransmitted up to $\tau$ times, the progress should be examined against the receiver-connected set $\Omega_0$ not the pool of all potential relay nodes.

MPR in (11) indicates the average radius of the outage-free multicast coverage for a given circular multicast cluster of radius $s$. Therefore, MPR on average represents the perfect cluster size. The following proposition provides a closed-form for $\eta$.

***Proposition** 1:* In a multicast PCP-based wireless network

$$\eta \;=\; s - \int_0^s e^{-2\pi\lambda_r \int_l^s r(1-(\mathbb{E}_I[F_H(\beta r^\alpha I_0)])^\tau)dr} dl, \quad (12)$$

where $\lambda_t$ is the source node density, $s$ is the cluster radius, $\lambda_r$ is the destination node density in a cluster and $\tau$ is the maximum number of retransmissions.

*Proof:* Similar to Section III here we also assume independent fading fluctuations among transmitters and receivers as well as negligible spatial correlation of the received interferences in different time slots. Therefore, in each retransmission attempt, destination nodes in cluster are identically and independently marked by $\mathbb{P}\{\text{SIR} \ge \beta\}$ [14], [18]. Accordingly, it is safe to assume that for an arbitrary destination node $Y_j$ in $\Omega_0$, it is independently marked by

$$\begin{aligned}
\mathbb{P}\{\delta_j > 0\} &= 1 - \mathbb{P}\left\{\sum_{t=1}^{\tau} \mathbf{1}\left(\text{SIR}_j(t) \ge \beta\right) = 0\right\}\\
&= 1 - \mathbb{P}\left\{\text{SIR}_j(t) < \beta, \forall t = \{1,\ldots,\tau\}\right\}\\
&= 1 - \prod_{t=1}^{\tau} \mathbb{P}\left\{\text{SIR}_j(t) < \beta\right\}\\
&= 1 - \left(\mathbb{P}\left\{\text{SIR}_j(t) \le \beta\right\}\right)^\tau\\
&\overset{(a)}{=} 1 - \left(\mathbb{E}_I[F_H(\beta r^\alpha I_0)]\right)^\tau, \; r \in [0, s], \quad (13)
\end{aligned}$$

where $(a)$ follows from the result of Eq. (36) in [14].

We define random variable $L = \max_{j\in\Omega_0} \|Y_j\|$. Its cumulative distribution function is

$$\begin{aligned}
\mathbb{P}\{L \le l\} &= \mathbb{E}\left[\prod_{Y_j\in\Omega_0} \mathbf{1}(\|Y_j\| \le l)\right]\\
&= \mathbb{E}\left[\prod_{Y_j\in\Omega_0} e^{\ln \mathbf{1}(\|Y_j\| \le l)}\right]\\
&= \mathbb{E}\left[e^{\sum_{Y_j\in\Omega_0} \ln \mathbf{1}(\|Y_j\|\le l)}\right]\\
&= e^{-2\pi\lambda_r \int_0^s (1-e^{-\ln \mathbf{1}(r\le l)})\mathbb{P}\{\delta_r>0\} r dr} \quad (14)\\
&= e^{-2\pi\lambda_r \int_l^s \mathbb{P}\{\delta_r>0\} r dr}, \;\; l \in [0, s]. \quad (15)
\end{aligned}$$

Substituting (13) into (15) yields

$$\mathbb{P}\{L \le l\} = e^{-2\pi\lambda_r \int_l^s r(1-(\mathbb{E}_I[F_H(\beta r^\alpha I_0)])^\tau)dr}. \quad (16)$$

MPR is

$$\eta = s - \int_0^s \mathbb{P}\{L \le l\} dl,$$

which gives the result after substituting (16). □

For a special case of Rayleigh fading (12) is reduced to

$$\begin{aligned}
\eta \;=\; s - \int_0^s \exp\Bigg(&-\frac{\lambda_r \beta^{-\frac{2}{\alpha}}}{\lambda_t \tilde{K}(\alpha)} \sum_{t=1}^{\tau} \binom{\tau}{t} \frac{(-1)^{t+1}}{t}\\
&\left(e^{-\pi\lambda_t \tilde{K}(\alpha)\beta^{\frac{2}{\alpha}} t l^2} - e^{-\pi\lambda_t \tilde{K}(\alpha)\beta^{\frac{2}{\alpha}} t s^2}\right)\Bigg) dl \quad (17)
\end{aligned}$$

For the general fading distribution, the following proposition provides an approximation of the MPR.

***Proposition** 2:* For a multicast PCP-based wireless network

$$\eta \simeq s - \int_0^s e^{-\pi\lambda_r(s^2-l^2)\sum_{m=1}^{\tau}\binom{\tau}{m}(-1)^{m+1}[1-\frac{(l^2+s^2)}{2\beta^{-\frac{2}{\alpha}}} m\pi\lambda_t\Delta(\alpha)]} dl,$$

where $\lambda_t$ is the source node density, $s$ is the cluster radius, $\lambda_r$ is the destination node density in a cluster, $\tau$ is the maximum number of retransmissions, and $\Delta(\alpha) = \mathbb{E}[H^{\frac{2}{\alpha}}]\mathbb{E}[H^{-\frac{2}{\alpha}}]$.

*Proof:* We first obtain $\mathbb{E}_I[F_H(\beta r^\alpha I_0)]$ by the following steps

$$\begin{aligned}\mathbb{E}_I[F_H(\beta r^\alpha I_0)] &= 1 - \mathbb{E}_I\left[\int_{\beta r^\alpha I_0}^{\infty} f_H(h)dh\right] \\ &= 1 - \mathbb{E}\left[\int_0^{\frac{H}{\beta r^\alpha}} f_I(x)\,dx\right] \\ &= 1 - \mathbb{E}\left[\mathbb{P}\left\{I_0 \le \frac{H}{\beta r^\alpha}\middle| H\right\}\right] \\ &\overset{(b)}{\approx} 1 - \mathbb{E}\left[e^{-\pi\lambda_t\beta^{\frac{2}{\alpha}}\mathbb{E}[H^{\frac{2}{\alpha}}]H^{-\frac{2}{\alpha}}r^2}\right], \end{aligned} \tag{18}$$

where the approximation $(b)$ follows from the dominant interferer approach proposed in [17]. In particular, for a realization $H = h$, transmitter $i$ is a dominant interferer if $\tilde{H}_i\|Y_i\|^{-\alpha} > hr^{-\alpha}/\beta$. Collecting all dominant interferers together, event $\{I_0 > hr^{-\alpha}/\beta\}$ surely occurs in the case when there is at least one dominant interferer.

Consequently, the probability of event $\{I_0 \le hr^{-\alpha}/\beta\}$ is upper-bounded by the probability that there is no dominant interferer, which is equal to $\exp(-\lambda_t \int_{\mathbb{R}^2} \mathbb{P}\{\tilde{H}\|Y\|^{-\alpha} > hr^{-\alpha}/\beta\}dY)$. Then, (18) is obtained after switching the integral to the polar coordination and performing some algebraic manipulation. Equation (18) is in fact a tight upper bound on $\mathbb{E}_I[F_H(\beta r^\alpha I_0)]$, thus can be considered as an approximation of $\mathbb{E}_I[F_H(\beta r^\alpha I_0)]$. Therefore,

$$\int_l^s r\left(1 - (\mathbb{E}_I[F_H(\beta r^\alpha I_0)])^\tau\right)dr \approx$$
$$\sum_{m=1}^{\tau}\binom{\tau}{m}(-1)^{m+1}\int_l^s r\left(\mathbb{E}\left[e^{-\pi\lambda_t\beta^{\frac{2}{\alpha}}\mathbb{E}[H^{\frac{2}{\alpha}}]H^{-\frac{2}{\alpha}}r^2}\right]\right)^m dr. \tag{19}$$

The integral in (19) can be extended as

$$\begin{aligned}&\int_l^s r\left(\mathbb{E}\left[e^{-\pi\lambda_t\beta^{\frac{2}{\alpha}}\mathbb{E}[H^{\frac{2}{\alpha}}]H^{-\frac{2}{\alpha}}r^2}\right]\right)^m dr \\ &= \int_l^s r\int_{h_1}\cdots\int_{h_m} e^{-\pi\lambda_t\beta^{\frac{2}{\alpha}}\mathbb{E}[H^{\frac{2}{\alpha}}]\sum_{n=1}^m h_n^{-\frac{2}{\alpha}}r^2}\prod_{n=1}^m dF(h_n)dr \\ &= \int_{h_1}\cdots\int_{h_m}\prod_{n=1}^m dF(h_n)\times \\ &\frac{e^{-\pi\lambda_t\beta^{\frac{2}{\alpha}}\mathbb{E}[H^{\frac{2}{\alpha}}]\sum_{n=1}^m h_n^{-\frac{2}{\alpha}}l^2} - e^{-\pi\lambda_t\beta^{\frac{2}{\alpha}}\mathbb{E}[H^{\frac{2}{\alpha}}]\sum_{n=1}^m h_n^{-\frac{2}{\alpha}}s^2}}{2\pi\lambda_t\beta^{\frac{2}{\alpha}}\mathbb{E}[H^{\frac{2}{\alpha}}]\sum_{n=1}^m h_n^{-\frac{2}{\alpha}}} \\ &\overset{(c)}{\approx} \frac{1}{2}(s^2 - l^2)\int_{h_1}\cdots\int_{h_m}\prod_{n=1}^m dF(h_n)\times \\ &\left\{1 - \frac{1}{2}(l^2 + s^2)\pi\lambda_t\beta^{\frac{2}{\alpha}}\mathbb{E}[H^{\frac{2}{\alpha}}]\sum_{n=1}^m h_n^{-\frac{2}{\alpha}}\right\}, \end{aligned} \tag{20}$$

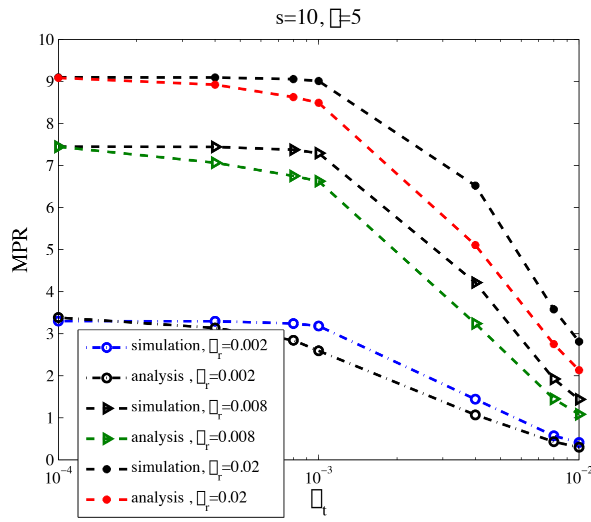


Fig. 2. MPR *vs.* $\lambda_t$ for different values of $\lambda_r$.

where the approximation $(c)$ follows from the second-order Taylor expansion for the exponential function along with some straightforward manipulations. Thus, $\int_l^s r\left(1 - (\mathbb{E}_I[F_H(\beta r^\alpha I_0)])^\tau\right)dr$ in (19) can be approximated by

$$\frac{(s^2 - l^2)}{2}\sum_{m=1}^{\tau}\binom{\tau}{m}(-1)^{m+1}\left(1 - \frac{(l^2 + s^2)}{2}m\pi\lambda_t\beta^{\frac{2}{\alpha}}\Delta(\alpha)\right), \tag{21}$$

where $\Delta(\alpha) \triangleq \mathbb{E}[H^{\frac{2}{\alpha}}]\mathbb{E}[H^{-\frac{2}{\alpha}}]$. Substituting (21) into (16) yields

$$\mathbb{P}\{L \le l\} \approx \exp\left\{-\pi\lambda_r(s^2 - l^2)\sum_{m=1}^{\tau}\binom{\tau}{m}(-1)^{m+1}\left[1 - \frac{1}{2}(l^2 + s^2)m\pi\lambda_t\beta^{\frac{2}{\alpha}}\Delta(\alpha)\right]\right\}.$$

Finally, substituting this into (12) (Proposition 1) completes the proof. □

## V. SIMULATIONS

Here our main objective is to examine the impact of important system parameters including, $\tau$, $s$, $\lambda_t$ and $\lambda_r$, on the MPR. The Monte-Carlo simulation set up is as follows. We generated a million snapshots of the network including the source and destined nodes. In each snapshot we randomly locate source nodes in a disk with radius 10000 meters. Then destined nodes are randomly scattered in disks with radius $s$ meters around each transmitter. Fading follows Rayleigh distribution with zero mean and unit variance.

Fig. 2 shows the MPR versus the density of transmitters in a multicast wireless network with cluster radius $s = 10$ and decoding delay $\tau = 5$. The result of our analysis closely follows the trend of the simulation results. Furthermore, by increasing $\lambda_t$, MPR decreases due to the increasingly deteriorating effect of the aggregated interference. For small values of $\lambda_t$, MPR is not changed by varying $\lambda_t$. Note that as the density of receivers

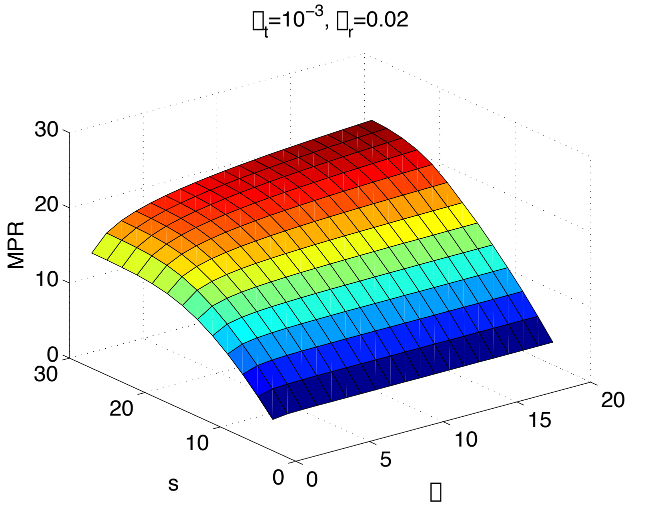


Fig. 3. MPR *vs.* $s$ and decoding delay $\tau$ for $\lambda_t = 0.001$, and $\lambda_r = 0.02$.

increases, MPR is also improved. The rate of improvement is fast for small values of $\lambda_r$, but slow for large enough $\lambda_r$.

Fig. 3 shows MPR versus cluster radius and decoding delay for $\lambda_t = 10^{-3}$, and $\lambda_r = 0.02$. For the case of $\tau = 1$, a cluster with radius $s/2$ is shown to be a perfect design choice, and hence the destined nodes located farther than $s/2$ will likely suffer higher probability of multicast outages. Furthermore, retransmissions are observed to enhance MPR, particularly for rather large values of $s$.

## VI. Conclusion

In this paper, we investigated the performance of a multicast network constituting of many coexisted clusters. It was shown that in general the network's performance is reluctant to increasing umber of retransmissions, although a small number of retransmissions is capable of boosting the system outage performance. We further defined and analysed the notion of multicast progress radius (MPR) for a given delay constraint. MPR measures the outage-free radius and indicates the percentage of a cluster that decode the packet for an affordable delay. We derived a closed-form for the MPR. By simulations we confirmed our analysis and studied the impact of several system parameters on the MPR.

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