Damage Assessment for Optimal Rollback Recovery

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Abstract—Conventional schemes of rollback recovery with checkpointing for concurrent processes have overlooked an important problem: contamination of checkpoints as a result of error propagation among the cooperating processes. Error propagation is unavoidable due to imperfect detection mechanisms and random interprocess communications, and it could give rise to contaminated checkpoints which, in turn, result in unsuccessful rollbacks. To counter the problem of error propagation, a damage assessment model is developed to estimate the correctness of saved checkpoints under various circumstances. Using the result of damage assessment, determination of the “optimal” checkpoints for rollback recovery—which minimize the average total recovery overhead—is formulated and solved as a nonlinear integer programming problem. Integration of damage assessment into existing recovery schemes is also discussed.

Index Terms—Damage assessment, error propagation, rollback recovery, checkpointing, nonlinear integer programming.

1 INTRODUCTION

Considerable research effort has been directed toward rollback error recovery with checkpointing for concurrent processes [1], [2], [3], which requires each process to save its intermediate state, called a checkpoint, several times during the execution so that, upon detection of an error, it may roll back to, and resume execution from, one of the saved checkpoints. A major problem in such rollback recovery is the rollback propagation (or domino effect) that results from uncoordinated/asynchronous checkpoints and/or random communications among the concurrent processes [4], [5], [6]. Two types of solution have been proposed: synchronous and optimistic approaches. A synchronous approach eliminates rollback propagation by synchronizing both checkpoint establishments and interprocess communications among the concurrent processes, whereas an optimistic approach minimizes the effect of rollback propagation by recording interprocess messages (message logging) and replaying them during rollback recovery.

The numerous synchronous approaches proposed over the last decade or so include rollback propagation detection [7], [8], redundant recovery points insertion [9], [10], two-phase commitment protocols [11], [12], [13], and pseudorecovery points [14]. The main disadvantage of these approaches is the coordination overhead incurred during normal operation, but their advantage is fast rollback recovery upon detection of an error.

Most optimistic approaches [3], [15], [16], [17], [18], [19], [20] assume optimistic message logging in which each process establishes its own checkpoints independently and saves the checkpoints and the received messages asynchronously with others. Rollback propagation in these schemes may originate from those messages which have not been recorded by the receiver processes because they need to be regenerated, thus causing the sender processes to roll back. Rollback propagation could also occur if the system rolled back to an inconsistent system state where some messages have been recorded and will be replayed by the receiver processes but may not be generated again by the rolled-back sender processes due to the nondeterministic nature of distributed systems. In such cases, the receiver processes would have to roll back further until they reach a consistent system state. To identify consistent system states, a dependency vector is attached to every message indicating the state of the sender process at the time of message transmission. Whenever a recorded message satisfies a certain criterion, it can be discarded to free storage space since the message won’t be needed for any future rollback recovery. Because these schemes do not eliminate rollback propagation completely, their disadvantage is a slower recovery than the synchronous approach in case an error is detected. Their advantage is the small overhead during normal operation, because checkpointing and message logging can be done asynchronously.

Besides rollback propagation, another major problem which has been overlooked in the existing approaches is error propagation as a result of random interprocess communications and incomplete detection coverage [21]. All the schemes mentioned above are based on the assumption that errors are detected immediately upon their occurrence. Under this assumption, the information contained in each checkpoint is always correct, because an error would otherwise have been detected before the checkpoint is saved. The benefits of this assumption (perfect detection coverage) are twofold: less secure storage space and less recovery overhead. A checkpoint can be discarded immediately as soon as the most recent consistent recovery line goes past it.

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However, it is practically impossible to achieve perfect coverage of error detection. The penalty for making such an unrealistic assumption is the global restart—restart from the very beginning—required when a saved checkpoint is found to be incorrect. This procedure is acceptable only if the global restart is not very expensive and/or the cost of keeping several checkpoints is very high.

There are, however, a class of applications where the global restart is very expensive and the cost of keeping more than one checkpoint is low. For such applications, one must minimize the occurrence of global restarts by keeping multiple checkpoints. Under imperfect error detection, the key issues are to assess damages (caused by errors before their detection) and determine “optimal” rollback points using the results of damage assessment.

Damage assessment refers to the evaluation of the correctness of checkpoints in each process. Whether a checkpoint is correct or not can never be known for sure until it is actually used for error recovery. But, it is possible to estimate the time when a process became contaminated based on the information obtained from error detection and fault diagnosis mechanisms. The probability of a checkpoint being incorrect can then be determined under the assumption that the checkpoints established in a process after it became contaminated are incorrect. We develop a new method for damage assessment on the basis of our earlier results on error propagation [21] and fault diagnosis [22], [23].

Because of the probabilistic nature of damage assessment, the success of any rollback recovery cannot always be guaranteed and, thus, other recovery mechanisms, such as the global restart, should always be provided in case of unsuccessful rollbacks. Using the result of damage assessment, one can determine the optimal rollback points by minimizing the average recovery overhead, which is based on the overheads of the rollback recovery and the global restart, and the probabilities of incorrect checkpoints.

The paper is organized as follows. In Section 2, damage assessment is carried out by deriving the distributions of contamination times of individual processes. Three different cases are considered for this derivation. In Section 3, the problem of determining the optimal rollback points is formulated and solved as a nonlinear programming problem. In Section 4, we discuss how to integrate damage assessment into an existing optimistic scheme of rollback recovery. The paper concludes with Section 5.

2 Damage Assessment

In this section, a method of damage assessment is developed by probabilistically characterizing the interval between the occurrence and the detection of an error using parameters associated with faults and errors.

A fault is defined as any defect capable of causing potential damage, or any deviation from the normal state of a computing system. An error is deviation from the specification of a program running on a computing system. Consider a multimodule computing system, where the processes communicate with one another via message passing. Each process is assumed to run on a separate module and, thus, the term “module” will mean a hardware module or the process running on it. The computing system can be represented by a digraph, $D = (V, E)$, where $V = \{v_1, ..., v_N\}$ denotes the set of nodes, and $E = \{e_{ij}, 1 \leq i, j \leq N\}$ denotes the set of directed edges. Each node in $V$ represents a module in the system, and a directed edge $e_{ij}$ represents the communication channel via which $v_i$ can send messages to $v_j$. A module is said to be faulty if it contains faults, and contaminated if it contains errors. Let $T_i^P$, the $v_i$’s faulty time, denote the time instant a fault occurs in $v_i$. Let $T_i^C$, the $v_i$’s contamination time, denote the time instant the first error occurs in $v_i$ as a result of either the manifestation of a fault within $v_i$ or the propagation of error(s) from other module(s). Then, damage assessment can be viewed as the estimation of the distributions of all modules’ contamination times.

Faults are detected directly by periodic diagnostics or fault detection mechanisms, such as self-checking circuits. Errors, on the other hand, are detected by error detection mechanisms, such as acceptance tests, capability checks, and time-outs. Upon detection of an error, a fault location procedure is called for to identify the faulty module. (Note that, in the absence of error propagation, the fault location procedure is not needed.) In the discussions below, we assume that there is only one faulty module to simplify mathematical derivation. Based on when and how a detection is made, damage assessment is carried out separately in the following three cases:

Case 1: An error is detected and the faulty module is identified.

Case 2: An error is detected but the faulty module is not yet identified.

Case 3: A fault is detected by periodic diagnostics.

When a fault is detected by a self-checking circuit, instruction retry is a better recovery scheme than rollback [24], [25]. The accuracy of damage assessment depends on the information collected from detection and diagnosis mechanisms. This information includes the location of the faulty module and the error syndrome $S$, expressed as

$$S = \begin{bmatrix} v_{h_1}, t_{1,1}; \ldots; v_{h_s}, t_{s} \end{bmatrix},$$

where $v_{h_1}, ..., v_{h_s}$ are the modules which have detected error(s), and $t_{1,1}, ..., t_{s}$ are the times at which the respective modules detected their first error. The modules which have not detected any error will be denoted by $v_{\bar{h}_1}, ..., v_{\bar{h}_{s-1}}$. In Case 1, both the error syndrome and the faulty module are known. In Case 2, only the error syndrome is available. In Case 3, the faulty module is known, but the error syndrome is not.

Let $\Delta(t)$ denote the conditional probability that $v_i$’s contamination time is no later than $t$, given the information necessary for damage assessment, and let $\hat{\delta}(t)$ denote the density function of $\Delta(t)$. Damage assessment is actually the derivation of $\hat{\delta}(t)$, $1 \leq i \leq N$. These functions will be used in the next section to determine the optimal rollback points by minimizing the average total recovery time.
2.1 Modeling Error Propagation

We briefly describe an error propagation model which is instrumental in the derivation of $\delta_i(t)$'s. The parameters defined in this model are random variables with certain distributions unless explicitly defined otherwise. For a detailed account of this model, see [21] and [22].

The rate of fault occurrence in module $v_i$ is characterized by its fault cycle, denoted by $Y_{vi}$, which is the time interval between two consecutive fault arrivals at $v_i$. If $v_i$ is the faulty module, then the fault latency of $v_i$, denoted by $L_{vi}$, is the interval between $v_i$'s faulty time and contamination time.

Errors in one module can propagate to other modules via propagation paths, which are communication paths from the source to the destination with distinct intermediate modules. Errors propagating into an already contaminated module are assumed to have negligible effects on its error propagation property (see [21] for a justification of this assumption). The error propagation time of each path is obtained as follows: First, identify all propagation paths and calculate the error propagation time of each path. Then, $X_{ij}$ is the minimum propagation time among all the paths from $v_i$ to $v_j$. For example, for a system represented by graph D1 in Fig. 1,

$$X_{13} = \min (B_{12} + B_{23}, B_{14} + B_{45} + B_{53}, B_{14} + B_{45} + B_{52} + B_{23}, B_{12} + B_{24} + B_{45} + B_{53}).$$

The distributions of $X_{ij}$ can be derived from those of $B_{ij}$ systematically and efficiently, as shown in [21].

Based on the types of detection mechanism used, faults in a module are classified into three categories:

1) **FD-detectable** if they can be uncovered by signal-level fault detection mechanisms [26],
2) **PD-detectable** if they are not FD-detectable but are detectable by periodic diagnostics, and
3) **undetectable** if they are neither FD-detectable nor PD-detectable.

A signal-level fault detection mechanism has the property that faults are detected immediately upon their occurrence [26]. If a fault is detected in a module during periodic diagnostics, errors might already have been induced and propagated to other modules. An undetectable fault can be captured only during the fault diagnosis after the errors induced by this fault are detected by some detection mechanism. The probabilities for any fault to be FD-detectable and PD-detectable, denoted by $C^F_i$ and $C^P_i$, respectively, are assumed to be fixed and known.

Error detection in a module $v_i$ is characterized by $K_i$, the detection latency of $v_i$, defined as the time interval from the $v_i$'s contamination time to its detection time, which is the time the first error detection is made. The distribution of $K_i$ depends on the error detection mechanisms used in $v_i$.

![Fig. 1. System graph D1.](image-url)
\[ L_j(S, \tau) = \text{Prob}\{E_{b_1} = t_1 - \tau, \ldots, E_{b_{N_{k}}} = t_N - \tau, E_{w_1} > T - \tau, \ldots, E_{w_{N_{k}}} > T - \tau\} \]

where \( T \) is the current time instance at which damage assessment is done. Using the Bayes's equation, \( \delta(t) \) is derived as

\[ \delta_k(t) = \frac{L_k(S, t) f^C_k(t)}{\int_{t_k}^{T_k} L_k(S, t) f^C_k(t) dt} \]

To estimate the damage on module \( v_j, j \neq k \), first calculate \( L_j(S, \tau_k, \tau_j) \), the likelihood of \( S \) when \( T_k^C = \tau_k \) and \( T_j^C = \tau_j \). Obviously, \( L_j(S, \tau_k, \tau_j) = 0 \) if \( \min(t_1, t_j) < \tau_k < T^D_k \), and for \( T_k^D < \tau_k < \min(t_1, t_j) \),

\[ L_j(S, \tau_k, \tau_j) = \text{Prob}\{X_{b_j} = \tau_j - \tau_k, \ldots, X_{b_{N_{k}}} = \tau_N - \tau_k\} \]

where \( E_{b_j}(s) = X_{b_j}(s) + K_i \) and \( X_{b_j}^2(s) \) is the error propagation time from \( v_k \) to \( v_j \) under the condition that \( X_{b_j} = s \). In other words, \( X_{b_j}^2(s) \) can be viewed as a special case of \( X_{b_j} \) with two sources of errors. One source is \( v_k \), from which errors propagate to \( v_j \) via all possible paths between \( v_k \) and \( v_j \), except for those passing through \( v_i \). The other source is \( v_j \), which starts the propagation of error to \( v_j \) via all possible paths between \( v_j \) and \( v_i \) at \( s \) time units after \( v_k \) became faulty. This interpretation simplifies the evaluation of \( X_{b_j}^2(s) \). For example, in graph D1 of Fig. 1, \( X_{b_3}^2(s) \) can be evaluated as

\[ X_{b_3}^2(s) = \min\{B_{12} + B_{23} + s + X_{43}\} \]

With the knowledge of \( L_j(S, \tau_k, \tau_j) \) for all \( \tau_k \) and \( \tau_j \),

\[ \delta_j(t) = \frac{\int_{t_k}^{T_k} L_j(S, \tau_k, t) f^C_k(\tau_k) d\tau_k}{\int_{t_k}^{T_k} \int_{t_j}^{T_j} L_j(S, \tau_k, \tau_j) f^C_k(\tau_k) d\tau_k d\tau_j} \]

Note that the expressions of \( \delta(t) \) and \( \delta_j(t) \) have the same denominator.

As an example, consider a system represented by D1 in Fig. 1 and an error syndrome \( S = [v_2, 0] \). We carried out simulation to determine all the likelihoods and the functions \( \Delta_i \) for this syndrome under different assumptions of the faulty module. All random parameters, except \( B_i \)'s, are assumed to be exponentially distributed. \( B_i \) is assumed to have a bimodal normal distribution, i.e.,

\[ B_i = \begin{cases} \mathcal{N}(\mu_i, \sigma_i^2) & \text{with probability } \eta_i \\ \mathcal{N}(\mu_i^2, \sigma_i^2) & \text{with probability } 1 - \eta_i \end{cases} \]

where \( \mathcal{N}(\mu, \sigma) \) denotes a normally-distributed random variable with mean \( \mu \) and standard deviation \( \sigma \). The specification and derivation of \( K_i \)'s distribution were discussed in [22] and, thus, will not be repeated here. The parameters used in the simulation are tabulated in Tables 1 and 2, where all time variables are assumed to have the same unit with the current error detection time set to zero.

The details of the simulation were reported in [22]. Some results of \( \Delta_i \) are plotted in Figs. 2 and 3, where the faulty module is \( v_1 \) and \( v_2 \), respectively. In Fig. 3, \( \Delta_3 \) and \( \Delta_4 \) are almost identical because the respective parameters are identical and both \( v_3 \) and \( v_4 \) are only one-hop away from \( v_2 \). The same can be said about \( \Delta_1 \) and \( \Delta_2 \).

### Table 1: Distribution of Random Parameters for All \( i \) and \( j \)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Distribution</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y_i )</td>
<td>Exponential</td>
<td>Mean = 100,000</td>
</tr>
<tr>
<td>( L_i )</td>
<td>Exponential</td>
<td>Mean = 40</td>
</tr>
<tr>
<td>( B_{ij} )</td>
<td>Bimodal Normal</td>
<td>( \eta_{ij} = 0.6 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \mu_{ij} = 40, \sigma_{ij} = 25 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \mu_{ij}^2 = 80, \sigma_{ij}^2 = 60 )</td>
</tr>
</tbody>
</table>

### Table 2: Parameters for the Faulty Module \( v_k \)

<table>
<thead>
<tr>
<th>Module</th>
<th>( C_s^t )</th>
<th>( C_h^t )</th>
<th>( T_s^c )</th>
<th>( T_h^c )</th>
<th>( T_s^d )</th>
<th>( T_h^d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4</td>
<td>0.4</td>
<td>-100,000</td>
<td>-50,000</td>
<td>-100,000</td>
<td>-50,000</td>
</tr>
<tr>
<td>2</td>
<td>0.4</td>
<td>0.4</td>
<td>-100,000</td>
<td>-40,000</td>
<td>-100,000</td>
<td>-40,000</td>
</tr>
<tr>
<td>3</td>
<td>0.4</td>
<td>0.4</td>
<td>-100,000</td>
<td>-30,000</td>
<td>-100,000</td>
<td>-30,000</td>
</tr>
<tr>
<td>4</td>
<td>0.4</td>
<td>0.4</td>
<td>-100,000</td>
<td>-20,000</td>
<td>-100,000</td>
<td>-20,000</td>
</tr>
<tr>
<td>5</td>
<td>0.4</td>
<td>0.4</td>
<td>-100,000</td>
<td>-10,000</td>
<td>-100,000</td>
<td>-10,000</td>
</tr>
</tbody>
</table>

#### 2.3 Damage Assessment for Case 2

In this case, damage assessment must be made without any knowledge on the location of the faulty module. One example is when error recovery and fault location are carried out in parallel. Another example is when the fault diagnosis routine fails to locate the faulty module due to either the occurrence of a transient fault or insufficient coverage of the diagnosis routine.

Let \( \pi_i \) represent the \( i \)'s faulty probability without considering the error syndrome. These \( \pi_i \)s are our subjective belief in locating the faulty module. If damage assessment is performed upon detection of an error, \( \pi_i \) is the same as the prior faulty probability \( \pi'_j \) determined by
where \( F_i^Y(T - T_i^Y) \prod_{j=1,j \neq i}^N (1 - F_j^Y(T - T_j^Y)) \) is our relative suspicion of \( v_i \) being the faulty module.

On the other hand, if damage assessment is performed after fault diagnosis, \( \pi_i \) is determined by

\[
\pi_i - \pi_i'(1 - \Phi_i)/\sum_{j=1}^N \pi_j'(1 - \Phi_j),
\]

where \( \Phi_i \) is the coverage of the diagnosis routine applied to \( v_i \). \( (1 - \Phi) \) can be interpreted as the likelihood of \( v_i \) being faulty but misdiagnosed to be nonfaulty. Using \( \pi_i \)'s as the weighting factors, \( \delta_j(t) \), \( 1 \leq j \leq N \), is derived as

\[
\delta_j(t) = \frac{\sum_{k=1}^N \pi_k \int_{t_k}^T L_k(S, \tau_k, t) \delta_k^T(\tau_k)d\tau_k}{\sum_{k=1}^N \pi_k \int_{t_k}^T L_k(S, \tau_k) \delta_k^T(\tau_k)d\tau_k}.
\]

### 2.4 Damage Assessment for Case 3

This case arises when a periodic diagnostic detects a fault in \( v_c \). It implies that the fault may have occurred any time between the present and the previous periodic diagnostic completed at \( T_k^p \). If the fault had occurred before \( T_k^p \), it would have been detected by the previous periodic diagnostic. Using the Bayes’ equation, the distribution of \( v_c \)'s faulty time in this case is derived as

\[
\delta_k^T(t) = \frac{\int_{t_k}^T f_k^Y(t - T_k^Y)(1 - F_k^Y(T - t))d\tau_k}{\int_{t_k}^T (1 - F_k^Y(T - \tau))d\tau}.
\]

for \( T_k^p \leq t \leq T \).
where $T$ is the fault detection time. If $Y_i$ is exponentially distributed, $T^c_k$ can be shown to be distributed uniformly over the interval $[T^o_k, T]$. The function $f_k^{T^c}(t)$ is then calculated as the convolution of $f_k^{T^o}(t)$ and $f_k^{L}(t)$, since $T^c_k = T^o_k + L_k$.

Though no error syndrome is available in this case, the fact that no errors have been detected so far is useful information. Let $\mathcal{T}(T, \tau_k, \tau_j)$ be the likelihood of no error detection up to time $T$ under the condition that $T^o_k = \tau_k$ and $T^c_j = \tau_j$, where $\tau_k$ is the faulty module and $\tau_j \leq \tau_k$. It is easy to see that

$$\mathcal{T}(T, \tau_k, \tau_j) = \Pr\left[X_{\tau_k} = \tau_j - \tau_k, E_{\tau_k}(\tau_j - \tau_k) > T - \tau_k, \ldots, E_{\tau_k}(\tau_j - \tau_k) > T - \tau_k\right]$$

The function $\delta_k(t)$ for $T^o_k \leq t \leq T$ is, thus, derived as

$$\delta_k(t) = \frac{\mathcal{T}(T, \tau_k, \tau_j) f_k^{T^o}(t)}{\int_{T^o_k}^{T} \mathcal{T}(T, \tau_k, \tau_j) f_k^{T^o}(t) dt}$$

And, for $1 \leq j \leq N, j \neq k$,

$$\delta_j(t) = \frac{\int_{T^o_k}^{T} \mathcal{T}(T, \tau_k, \tau_j) f_k^{T^o}(t) dt}{\int_{T^o_k}^{T} \mathcal{T}(T, \tau_k, \tau_j) f_k^{T^o}(t) dt}$$

### 2.5 Remarks

The derivation of $\delta(t)$s in all three cases requires the knowledge of $f_k^{T^o}(t)$ and some likelihood functions such as $L_{ij}, L_{ij}', \text{ or } \overline{L}_{ij}$. It is very complex to derive these likelihoods analytically and, thus, these likelihood functions are usually calculated numerically. Since even the numerical solution requires an excessive amount of time, the likelihood functions are computed off-line. However, the function $\delta(t)$ has to be derived on-line since $f_k^{T^o}(t)$ depends upon on-line information, such as the elapsed time from a previous fault and the elapsed time from the previous periodic diagnostic. If the situation does not allow for on-line derivation of $\delta(t)$, $\delta(t)$ can be determined off-line using the noninformative prior function in place of $f_k^{T^o}(t)$. The noninformative prior function is usually a uniform density function, i.e., a faulty module could be contaminated any time prior to the detection time with an equal probability.

#### 2.5.1 List of Symbols

- $F_i^V(f_i^V)$: Distribution (density) function of the random variable $V_i$.
- $f_i^V$: Density function of the random variable $V_i$.
- $Y_i$: Fault cycle of $v_i$.
- $L_i$: Fault latency of $v_i$.
- $C_i^F$: Coverage of fault detection in $v_i$.
- $C_i^P$: Coverage of periodic diagnostics in $v_i$.
- $B_{ij}$: Direct propagation time from $v_i$ to $v_j$.
- $X_{ij}$: Error propagation time from $v_i$ to $v_j$.
- $K_i$: Detection latency in $v_i$.
- $H_i$: Hypothesis of $v_i$ being the faulty module.
- $\pi_i^{n}$: Prior (posterior) $\Pr[H_i]$.
- $S$: Fault syndrome.
- $\mathcal{L}(S)$: Fault syndrome’s likelihood function if $H_i$ is true.
- $E_i/	au$: Error latency for the error originated from $v_i$ and detected in $v_i$.
- $T_i^T$: Last time a repair was done on $v_i$.
- $T_i^D$: Last time a thorough diagnostic was applied to $v_i$.
- $T_i^P$: Last time a periodic diagnostic was applied to $v_i$.
- $T_i^C$: The contaminating time of $v_i$.
- $T_i^F$: The faulty time of $v_i$.
- $\Phi$: Percentage of faults that the diagnosis routine can uncover in $v_i$ and $\phi = d\Phi/dt$.

### 3 Optimal Rollback Points

In this section, we will determine the optimal rollback points for rollback recovery using the results from damage assessment.

Let $T$ denote the current time. For the convenience of problem formulation, $T$ is designated as the origin of the time axis and all events are enumerated backward from $T$. For example, the $k$th checkpoint (message) means the $k$th previous checkpoint (message) from $T$. Let $c_p^k$ be the time $v_i$ established the $k$th checkpoint, and $r_i^k$ be the time $v_i$ received the $k$th message. If $\tau_i$ rolls back to the $k$th checkpoint, then $v_i$’s rollback distance is defined as

$$d_i(k) = T - c_p^k.$$  

The probability that $\tau_i$ can successfully roll back to the $k$th checkpoint is denoted by $p_i(k)$. Note that $p_i(k)$ is the same as the probability that $v_i$ is contaminated after $c_p^k$. Hence,

$$p_i(k) = \int_{c_p^k}^{\infty} \delta_i(\tau) d\tau = 1 - \Lambda_i(c_p^k).$$

The above equation holds only when $v_i$ is the faulty module. If $v_i$ is not the faulty module, its contamination is
caused by incoming message(s) from other module(s), i.e., error propagation. Since \( v_i \) receives messages only at \( r_i^n \), if \( v_i \) is not the faulty module and \( r_i^n < c_{pi}^k < r_i^{n+1} \), then 
\[
p_i(k) = 1 - \Delta_i(r_i^n).
\]

There is one exception, however; if an error has been detected in \( v_i \) and \( v_i \) has received no messages since \( c_{pi}^k \), then 
\[
p_i(k) = 0.
\]

Define the overhead of rollback recovery as the sum of the rollback distances \( d_S \) in all modules, which is the total computation to be redone. Our object is to minimize the mean recovery overhead, denoted by \( O \). Let \( Z \) denote the total overhead for a global restart which will be invoked in case rollback recovery fails. Rollback recovery will eventually fail if any module of the system rolls back to an incorrect checkpoint. Thus, the probability of successful rollback recovery is the product of the probability that each module’s rollback point is correct. Hence, \( O \) can be expressed as
\[
O = \left( \prod_{i=1}^{N} p_i(k_i) \right) \sum_{i=1}^{N} d_i(k_i) + \left( 1 - \prod_{i=1}^{N} p_i(k_i) \right) \left( \sum_{i=1}^{N} d_i(k_i) + Z \right)
\]
\[
= \sum_{i=1}^{N} d_i(k_i) + \left( 1 - \prod_{i=1}^{N} p_i(k_i) \right) Z,
\]
where \( k_i \) is a nonnegative integer indicating the rollback point of \( v_i \). There is an upper limit, say \( C_i \), for \( k_i \), \( 1 \leq i \leq N \), because each secure storage will have a limited capacity. The optimal rollback problem is then stated as follows:

Minimize \[
\sum_{i=1}^{N} d_i(k_i) + \left( 1 - \prod_{i=1}^{N} p_i(k_i) \right) Z
\]
subject to \( k_i \leq C \), for \( 1 \leq i \leq N \).

This is a nonlinear integer programming problem.

Integer programming problems are in general very difficult to solve. Most of existing algorithms, such as cutting plane methods and enumerative search methods, are applicable only if the objective function is a linear function. To develop an algorithm for a nonlinear integer programming problem, we have to find and utilize some special properties of the object function \( O \). One useful property is the convexity of \( O \). If \( p_i \) is expressed as in (3.2), \( O \) can be viewed as a continuous function of \( d_S \). The following lemma then provides the necessary and sufficient condition for the convexity of \( O \).

**Lemma 1.** \( O \) is convex with respect to \( d_i \) if and only if \( \delta(t), t = T - d_i, \) is monotonically increasing.

**Proof.** From (3.3),
\[
\frac{dO}{dt} = 1 - \frac{\delta p_i}{\delta t} \sum_{j=1,j\neq i}^{N} p_j
\]
and
\[
\frac{d^2O}{dt^2} = -\frac{\delta^2 p_i}{\delta t^2} \sum_{j=1,j\neq i}^{N} p_j
\]
\(O\) is convex with respect to \( d_i \) if and only if \( \frac{d^2 p_i}{dt^2} < 0 \), since both \( Z \) and \( p_j \) are positive. From (3.1) and (3.2),
\[
\frac{\delta p_i}{\delta t} = -\frac{\delta p_i}{\delta t} \frac{\delta t}{\delta t} = \delta_i(t)
\]
and
\[
\frac{\delta^2 p_i}{\delta t^2} = -\delta_i(t).
\]

Thus, \( \frac{\delta^2 p_i}{\delta t^2} < 0 \) if and only if \( \delta_i(t) > 0 \), i.e., if \( \delta(t) \) is monotonically increasing.

In general, \( \delta(t) \) may not be monotonically increasing for \( t \in [-\infty, \infty] \). But, there must exist \( T \), such that \( \delta(t) \) is monotonically increasing for \( t \in [-\infty, T] \), since \( \delta(t) \to 0 \) as \( t \to -\infty \).

Based on the convexity of \( O \), the following algorithm is developed to solve the optimal rollback problem.

**Algorithm RB:**

1. For \( 1 \leq i \leq N \), let \( k_i \) be the smallest integer with the property \( p_i^{k_i} < T_i \).

Let \( k_i := C_i \) if \( k_i > C_i \) and \( Y := \prod_{i=1}^{N} p_i(k_i) Z \).

2. Let \( S := \{ i : k_i < C_i \} \).

If \( k_i = C_i \) \( \forall i \in S \), then go to Step 6.

3. \( \forall i \in S \), let \( y_i := \delta d_i - (\rho_i - 1) Y \) where
\[
\delta d_i := d_i(k_i + 1) - d_i(k_i),
\]
\[
\rho_i := 1 + \frac{\delta p_i}{\prod_{j=1,j\neq i}^{N} p_j}
\]
\[
\phi_i := p_i(k_i + 1) - p_i(k_i).
\]

4. If \( y_i \geq 0 \) \( \forall i \in S \), then go to Step 5.

Otherwise, \( \forall i \in S \), let \( k_i := k_i + 1 \) and \( Y := \rho_i Y \).

Go to Step 2.

5. For any \( A \subset S \), let \( D(A) := \sum_{i \in A} \delta d_i - \prod_{i \in A} \left( \rho_i - 1 \right) Y \).

Find a set \( S^* \subset S \) such that \( D(S^*) < 0 \).

If such an \( S^* \) does not exist, then go to Step 6.

Otherwise, \( \forall i \in S^* \), let \( k_i := k_i + 1 \) and \( Y := \rho_i Y \).

Go to Step 2.

6. Terminate the algorithm.

The current value of \( k_i \), \( 1 \leq i \leq N \), is the optimal recovery point for \( v_i \).

**RB** is essentially an algorithm which searches from smaller \( k_i \)s toward larger \( k_i \)s. In Step 1, \( k_i \) is initialized to the smallest value that will put the corresponding \( p_i^{k_i} \) inside the convex region of the function \( O \) with respect to \( k_i \). By Lemma 1, the optimal solution for \( k_i \) is obtained when \( O \) cannot be reduced any further by incrementing any subset of \( k_i \)s. If \( k_i \) reaches its limit \( C_i \) before the minimum \( O \) is reached, the optimal solution for \( k_i \) will be \( C_i \). If all \( k_i \)s reach their limits during the search, **RB** terminates immediately (Step 2).

The search in **RB** is conducted in two levels. In the first level (Steps 3 and 4), we check whether incrementing a
legitimate $k_i$ (i.e., $k_i < C_i$) would result in a smaller mean overhead. The set $S$ defined in Step 2 is the set of all legitimate $k_i$’s at that moment. The variable $y_i$ is actually the difference in $O$ with $k_i$ and $k_i + 1$, i.e.,

$$y_i = O(\ldots, k_i + 1, \ldots) - O(\ldots, k_i, \ldots).$$

If all $y$s are greater than zero, the next level of search is performed in Step 5. If at least one $y_i$ is negative, all $k$s with a negative $y_i$ will be incremented by one, which will yield an even smaller mean overhead because of the following lemma.

**Lemma 2.** Let $y_i, 1 \leq i \leq N$, be defined as in Step 3 of BB. If $y_i < 0$ and $y_j < 0$, then

$$O(\ldots, k_i + 1, \ldots, k_j + 1, \ldots) < O(\ldots, k_i, \ldots, k_j, \ldots).$$

**Proof:** Let $\delta d_i = d_i(k_i + 1) - d_i(k_i)$ and $\delta p_i = p_i(k_i + 1) - p_i(k_i)$ for $1 \leq i \leq N$. Then,

$$O(\ldots, k_i + 1, \ldots, k_j + 1, \ldots) = \sum_{i=1}^{N} d_i(k_i) + \delta d_i + \delta d_j + \left(1 - (p_i(k_i) + \delta p_i)(p_j(k_j) + \delta p_j) \prod_{n=1,n \neq i,j}^{N} p_n(k_n)\right)Z$$

$$= O(\ldots, k_i, \ldots, k_j, \ldots) + y_i + y_j - \delta p_i \delta p_j \prod_{n=1,n \neq i,j}^{N} p_n(k_n)Z$$

$$< O(\ldots, k_i, \ldots, k_j, \ldots),$$

since $y_i < 0, y_j < 0$, and $\delta p_i \delta p_j \prod_{n=1,n \neq i,j}^{N} p_n(k_n)Z > 0$. \(\square\)

Even if $O$ cannot be reduced any further by incrementing any single legitimate $k_i$, it is still possible to reduce $O$ further by incrementing a subset of $S$. If such a subset (i.e., $S^*$) exists, it would be found in Step 5. The function $D(A)$ in Step 5 is the difference in $O$ with $k$s and $k_i + 1$s, $\forall i \in A$. Finding an $S^*$ is not a trivial task because the number of subsets increases exponentially with the number of legitimate $k$s. A branch-and-bound (B&B) algorithm is, thus, developed below to either find an $S^*$, if it exists, or to show that it does not exist.

Let $S_n(A)$ represent any set with $n$ elements which is both a subset of $S$ and superset of $A$. In other words, $\|S_n(A)\| = n$ and $A \subset S_n(A) \subset S$, where $\|X\|$ denotes the cardinality of the set $X$. The elements of $S$ will be sorted in two lists: one based on $\delta d_i$ and the other based on $\rho_i$ so that the sets defined below can be easily determined. For any $A \subset S$ and $n \leq \|S\|$, let $S_{\text{max}}^n(A, n)$ be defined such that

1. $A \subset S_{\text{max}}^n(A, n)$ and $\|S_{\text{max}}^n(A, n)\| = n$,
2. $\rho_i \geq \rho_j$ for any $i \in S_{\text{max}}^n(A, n) - A$ and $j \in S - S_{\text{max}}^n(A, n)$,

and let $S_{\text{min}}^n(A, n)$ be defined such that

1. $A \subset S_{\text{min}}^n(A, n)$ and $\|S_{\text{min}}^n(A, n)\| = n$,
2. $\delta d_i \leq \delta d_j$ for any $i \in S_{\text{min}}^n(A, n) - A$ and $j \in S - S_{\text{min}}^n(A, n)$.

For $A \subset S$ and $n \geq \|A\|$, define

$$w(A, n) = \sum_{i \in S_{\text{min}}^n(A, n)} \delta d_i - \left( \prod_{i \in S_{\text{min}}^n(A, n)} \rho_i - 1 \right)Y,$$

and it can be shown easily that $w(A, n) \leq D(S_n(A))$. With the above definitions, we now present the following B&B algorithm.

**Algorithm BB:**

1. Let $n := \|S\|$.
2. If $n = 1$, the algorithm terminates and it is concluded that $S^*$ does not exist.
3. Let $A_n := S_{\text{min}}^n(\emptyset, n) \cap S_{\text{max}}^n(\emptyset, n)$, where $\emptyset$ denotes the empty set.
4. If $w(A_n, n) \geq 0$, let $n := n - 1$ and go to Step 2.
5. If $\|A_n\| = n$, the algorithm terminates with $S^* := A_n$.
6. Examine all subsets of $S - A_n$ with $n - \|A_n\|$ elements using a depth-first search to find a set $R$ with $D(A_n \cup R) < 0$.

Whenever such an $R$ is found, the algorithm terminates with $S^* := A_n \cup R$.

Otherwise, let $n := n - 1$ and go to Step 2.

The basic strategy in finding an $S^*$ is to classify subsets of $S$ based on the cardinality of each subset and, then, search within each subset class for a possible solution. The subset class with the most number of elements is searched first. If no solutions are found in all subset classes, we conclude that $S^*$ does not exist. In searching the subset class with $n$ elements, $A_n \subset S^*$ can be assumed to shorten the search according to the following lemma.

**Lemma 3.** If an $S^*$ exists and $\|S^*\| = n$, then there must exist a set $S'$ with $n$ elements such that $D(S') < 0$ and $A_n = S_{\text{min}}^n(\emptyset, n) \cap S_{\text{max}}^n(\emptyset, n) \subset S'$.

**Proof:** If $A_n \subset S^*$, the proof is trivial since we can let $S' = S^*$. If $A_n \not\subset S^*$, then, by the definitions of $S_{\text{min}}^n$ and $S_{\text{max}}^n$, we can find a set $A' \subset S^*$ with $\|A'\|$ elements such that $\sum_{i \in A} \delta d_i \leq \sum_{i \in A'} \delta d_i$ and $\prod_{i \in A} \rho_i \geq \prod_{i \in A'} \rho_i$.

The lemma is proved by letting $S' = (S^* - A') \cup A$, since

$$0 > D(S^*) = \sum_{i \in S^* - A'} \delta d_i \sum_{j \in A'} \delta d_j - \left( \prod_{i \in S^* - A'} \rho_i \prod_{j \in A'} \rho_j - 1 \right)Y$$

$$\geq \sum_{i \in S^* - A'} \delta d_i \sum_{j \in A'} \delta d_j - \left( \prod_{i \in S^* - A'} \rho_i \prod_{j \in A'} \rho_j - 1 \right)Y$$

$$= D((S^* - A') \cup A).$$

Since $w(A_n, n)$ is the lower bound of $D(S_n(A_n)) \forall S_n(A_n)$, $S^*$ cannot exist in the subset class with $n$ elements if $w(A_n, n) \geq 0$. This property will be applied during the depth-first search to minimize the search time. To illustrate this, suppose $n = 5, \|A_n\| = 2$, and $S - A_n = \{1, 2, 3, 4, 5\}$. The objective is to
find a set $R$ with 3 elements and $D(A_v \cup R) < 0$. The depth-first search tree in this case is shown in Fig. 4, where the subscript of $R$ denotes the search sequence. If for any non-leaf set $R_i$ in the tree $w(A_v \cup R_i, n) \geq 0$, then it is not necessary to search the children sets of $R_i$. For example, if $w(A_v \cup R_1, n) \geq 0$, then the next set to examine would be $R_{10}$, skipping all the sets between them.

We now present an example of the optimal rollback recovery problem using the results of damage assessment in Figs. 2 and 3.

**EXAMPLE.** Consider the system represented by $D_1$ in Fig. 1. Let $S = \{v_2, 0\}$ and the total global restart overhead be 1,200. The rollback distances of all recovery points are tabulated in Table 3. We assume that there are only six recovery points for each module, so $C_i = 6 \forall i$. Each module’s probability of successful rollback is obtained from Figs. 2 and 3. Results obtained from applying Algorithms $RB$ and $BB$ are summarized in Table 4.\(^1\) Note that if the faulty module and the error-detecting module are farther apart, the mean overhead will generally be higher because modules’ contaminating probabilities are usually larger. Another interesting observation is that we have yet to discover a case where $BB$ finds an $S^\dagger$. It appears that, in most cases, if $O$ cannot be reduced by incrementing any single legitimate $k_i$, it will not be reduced by incrementing a subset of $k_i$. Therefore, using the bounding function $w(A, n)$ becomes very beneficial, since it can speed up the termination of $BB$.

The optimal recovery points obtained in the above example produce a much larger total rollback distance than what other optimistic approaches would have produced. But, in view of the facts that

1) detection mechanisms are imperfect, and  
2) checkpoints that may have been contaminated are considered,

our solution should outperform other optimistic approaches in the sense of minimizing the mean recovery overhead.\(^2\) Actually, our solution will be identical to an optimistic solution if the detection mechanisms in every module are perfect.

### 4 Damage Assessment for Optimistic Rollback

Many rollback recovery schemes have been proposed for distributed systems over the last decade or so, but none of them have considered corrupted checkpoints and damage assessment. Thus, the focus of this section is to show how to integrate damage assessment with existing checkpointing schemes, especially the optimistic message logging and checkpointing schemes such as the one proposed by Johnson and Zwaenepoel [18].

In general, any rollback recovery scheme equipped with damage assessment should comply with the following steps after detecting an error.

1) **Damage assessment:** Using the algorithms for determining the optimal rollback points as derived in the previous section, compute a set of checkpoints which are safe to rollback to, while considering the overall overhead.

2) **Cancellation of corrupted messages:** All checkpoints established after the set of safe checkpoints are considered to have been corrupted. Hence, all messages sent after a corrupted checkpoint are also corrupted and should be removed from the receiver’s stable storage if they have been logged. Furthermore, all messages sent after the receipt of a corrupted message are considered corrupted and must be canceled as well. The messages sent between a safe checkpoint and its next checkpoint are considered correct if they have not been canceled. In some rare events, rollback may propagate from a safe checkpoint if one of the messages received before the establishment of the safe checkpoint has been canceled.

3) **Search for a recoverable system state:** The system will likely be in a recoverable state after canceling all corrupted messages, since most messages would have been logged if they can survive the cancellation. If not, a new recoverable system state must be determined from the current state.

4) **Recovery:** Roll back to the derived recoverable system state. The system will recover successfully if this system state is indeed not corrupted as predicted in the damage assessment.

In Johnson and Zwaenepoel’s scheme [18], a search algorithm, $FIND\_REC$, based on a known recoverable state, is proposed to check if a new state developed afterwards is a recoverable system state. The initial state of the system is obviously recoverable. Since then, whenever the system

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1. The data for the case of $v_5$ being the faulty module are obtained by simulation similarly to the other two cases.  
2. Direct comparison between our solution and others is not possible, though, because none of the others considered these two facts.
moves into a new state, _FIND_REC_ is called in to determine if it is recoverable. If the new state is recoverable, it becomes the latest recoverable system state, replacing the previous one. Thus, a series of recoverable system states will be derived during normal operation and can be stored in stable storage. Later, when an error is detected, this knowledge can greatly simplify the search for the latest recoverable system state before the current state is established by damage assessment and message cancellation. The _FIND_REC_ algorithm can be invoked here using one of known past recoverable system states closest to the current state as the base.

Damage assessment can be integrated differently into an existing recovery scheme. However, the problem of finding the optimal checkpoints in the previous section will have to be reformulated under different assumptions and constraints. For example, a more efficient damage assessment for Johnson and Zwaenepoel’s scheme [18] can take advantage of the series of recoverable system states derived during normal operation. Instead of rolling back to any combination of checkpoints, we need to evaluate only the success rate and the overhead of rolling back to these past recoverable system states. The optimization is much simpler and there is no need for canceling messages and searching for a new recoverable state.

The main difficulty of integrating damage assessment into an existing rollback recovery scheme is the size of stable storage. Without considering error propagation, whenever a checkpoint becomes part of a recoverable system state, all messages received before the checkpoint can be safely discarded from the stable storage. With damage assessment, however, checkpoints and messages must be kept in the stable storage for a longer period, thus requiring a larger stable storage. This is the cost one has to pay instead of using more expensive (both in time and resource) error detection mechanisms to enhance the coverage and thus remove the need for damage assessment.

### 5 Conclusion

Checkpointing and rollback recovery for concurrent processes have received considerable attention. Two crucial problems which must be resolved are the problems of rollback propagation (or the domino effect) and error propagation. All previous work focused on solving the rollback propagation problem and avoided the error propagation problem by assuming detection mechanisms to be perfect so that all checkpoints are correct. However, it is practically impossible to make error detection perfect and, thus, a checkpoint may be incorrect if it is established after the process became erroneous. These contaminated checkpoints will lead to unsuccessful rollback recovery. In this paper, we have developed a method of damage assessment to handle the error propagation problem, based on our earlier work on error propagation and fault location. Using the results of damage assessment, we formulated and solved the problem of determining the optimal rollback points by minimizing the average total recovery overhead. Integration of damage assessment with existing recovery schemes is discussed using an optimistic message logging and checkpointing scheme as an example.

It is difficult to compare our approach directly with the previous work because the assumptions and the performance criteria are quite different. All previous work assumes perfect acceptance tests so that error propagation is bounded by the recovery points and the success of rollback recovery is guaranteed if proper rollback points are selected. The performance of a rollback recovery scheme is judged by the overhead of a successful rollback. By contrast, we assume that rollback recovery may fail under certain circumstances since any recovery point could be contaminated by error propagation due to imperfect coverage of acceptance tests. We have to resort to an alternative recovery scheme, such as a global restart should rollback recovery fail. The rollback points determined in this paper is thus to minimize the expected overhead taking into consideration the overhead of the rollback recovery and the alternative scheme, and the probability of successful rollback recovery.

The damage assessment model developed in this paper can also be used to solve other problems, such as the scheduling of periodic diagnostics and the determination of optimal inter-checkpoint intervals. These problems are matters of our future inquiry.
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