Comment on 
“A Pre-Run-Time Scheduling Algorithm for Hard Real-Time Systems”

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Abstract—In Shepard and Gagne [1], a branch-and-bound implicit enumeration algorithm is described whose purpose is to generate a feasible schedule, if any, for each processor on a multiprocessing node running hard real-time processes. The optimization criterion is to minimize process lateness defined as the difference between the process completion time and deadline. We show in this correspondence that this algorithm does not always succeed in finding a feasible solution, and describe the reason why the algorithm might fail.

Index Terms—Pre-run-time scheduling, hard real-time systems, branch and bound scheduling algorithm, multiprocessors, precedence constraints.

1 INTRODUCTION

SHEPARD AND GAGNE [1] describe a pre-run-time branch-and-bound implicit enumeration algorithm, which attempts to find a feasible schedule for a set of hard real-time processes subject to precedence and exclusion constraints. Processes are assumed to be statically assigned to processors on a multiprocessing node.

Compared to ad hoc hand-crafted scheduling methods, the application of this pre-run-time scheduling algorithm to hard real-time systems should reduce the resources required for run-time scheduling and context switching. Unless the application violates its load specification, pre-run-time scheduling also provides a means to verify and guarantee the observance of timing constraints throughout the life cycle of the system. In addition, the scheduling model used in [1] provides a systematic and tractable approach to modifying application software. Namely, when the software is modified, the applicable scheduling model parameters may be altered, and a new set of run-time schedules can be generated. This approach eliminates the need for fine tuning the code via repeated stochastic simulations and may reduce the cost of software maintenance. An important contribution of [1] is applying their algorithm to a model of a real system, namely the F-18 Mission Computer Operational Flight Program [2]. Due to the importance of automated pre-run-time scheduling to the design and maintenance of hard real-time systems, we would like to clarify in this correspondence the fact that the algorithm in [1] is not guaranteed to find a feasible solution (where no process misses its deadline), when one exists. We also provide the reason why it occasionally fails.

The general idea of the branch-and-bound algorithm in [1] is to minimize schedule lateness defined as the maximum process latency over all scheduled processes. At each search vertex an earliest deadline first (EDF) schedule is found for each processor (subject to the defined constraints between processes), a process with the maximum latency (the latest process) is identified, and an attempt to reduce its latency is made. Shepard and Gagne [1] argue that

“to improve the lateness of the latest process in a schedule, that process must be forced to precede or preempt another process presently preceding it on the same schedule” (p. 673). This is accomplished by appropriately introducing an extra precedence or preemption constraint. A search vertex is expanded by generating a child vertex for each possible lateness improvement; that is, for each possible way the latest process may be forced to precede or preempt some earlier process on the same schedule (by introducing an extra constraint) such that schedule lateness is reduced. The algorithm terminates when a feasible solution is reached, or when no further improvements (over the nonfeasible schedules obtained thus far) are possible, i.e., no more vertices can be expanded. As will be demonstrated below, in the latter case a feasible solution may actually exist.

2 Counterexample

Consider a set of four processes \( T_1, T_2, T_3, \) and \( T_4 \), with a constraint \( T_2 \) precedes \( T_3 \). No exclusion constraints are defined (specifically, no exclusion constraint exists between \( T_1 \) and \( T_2 \)). Each process \( T_i \) has an arrival time \( r_i \), a computation time \( c_i \), and a deadline \( d_i \), as shown in Table 1. To account for precedence constraints, task arrival times and deadlines are modified as described in [1]. Thus, the arrival time \( r_2 \) becomes \( r_2 = 0 + c_3 = 2 \). Similarly, the deadline \( d_3 \) becomes \( d_3 = 12 - c_2 = 7 \).

<table>
<thead>
<tr>
<th>Task</th>
<th>( c_i )</th>
<th>( r_i )</th>
<th>( d_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_1 )</td>
<td>4</td>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>( T_2 )</td>
<td>5</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>( T_3 )</td>
<td>2</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>( T_4 )</td>
<td>3</td>
<td>0</td>
<td>6</td>
</tr>
</tbody>
</table>

Assume the system has two processors, \( P_1 \) and \( P_2 \). Let processes \( T_1 \) and \( T_2 \) be assigned to processor \( P_1 \), and processes \( T_3 \) and \( T_4 \) be assigned to processor \( P_2 \). Fig. 1a depicts the EDF schedule obtained at the root of the search. The latest process is \( T_2 \), which is scheduled after process \( T_1 \) on \( P_1 \). The schedule is not feasible because \( T_2 \) misses its deadline. According to [1], since \( T_1 \) has an earlier deadline than \( T_2 \), forcing \( T_2 \) to precede or preempt \( T_1 \) can only increase schedule lateness. Thus, the root vertex cannot be expanded and the algorithm terminates with no feasible solution. However, a feasible solution does exist! If process \( T_3 \) is scheduled before \( T_4 \) on \( P_2 \), all processes meet their deadlines as depicted in Fig. 1b. The resulting schedule, however, is no longer EDF.

The reason why the algorithm failed to find a feasible solution is that it attempts to reduce schedule lateness by modifying only the schedule of the processor running the latest task. In general, if the latest process has predecessors on other processors it is possible to improve lateness by shifting these predecessors earlier in their schedules. This aspect was apparently overlooked by the authors of [1].

It is conjectured that one way to fix this flaw is to extend the notion of a contiguous set, used in [1], to span over multiple processors. The contiguous set, as defined in [1], is the set of all processes immediately preceding the latest process on the same processor in a continuous execution sequence (including the latest process itself). The branching function considers rescheduling only the processes in the contiguous set, all of which are on the same processor with the latest process. A pre-run-time multiprocessor

1. To obtain the schedule in Fig. 1b, we also need to permit \( T_3 \) to preempt \( T_1 \) by introducing a corresponding preemption constraint [1].
2. The application of this idea to the algorithm in [1] was suggested by one of the referees reviewing an earlier version of this correspondence.

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Manuscript received 14 June 1995. Recommended for acceptance by D. Eager.
For information on obtaining reprints of this article, please send e-mail to: tse@computer.org, and reference IEEECS Log Number 106560.
A scheduling algorithm, which uses the extended notion of the contiguous set is proposed in Xu [3]. It defines it as a set $Z[l]$ of all processes in the period preceding and including the latest process such that within that period there does not exist any time where all processors are idle. This allows the branching function to modify schedules of other processors when rescheduling processes in the contiguous set. Further intuition behind this definition is described in [3].


Acknowledgment

The work reported in this paper was supported, in part, by the Advanced Research Project Agency (ARPA), monitored by the United States Air Force Rome Laboratory under Grant No. F30602-95-1-0044. Any opinions, findings, and conclusions or recommendations are those of the authors and do not necessarily reflect the views of the funding agency.

References

