# Susceptibility of Controller Computers to Environmental Disruptions and Its Effects on System Stability

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Abstract — ElectroMagnetic Interference (EMI) causes controller *upsets* manifested as control-law computation errors in digital computers or transmission disturbances between sensor and/or actuator lines. Though its effects are likely to be transient, frequent occurrences of these upsets may lead to the loss of system stability.

In this paper, we compute the stationary probability of those upsets induced due to EMI by using parameters accounting for EMI behavior and the conditional probabilities of upsets in the presence of EMI. The latter represents susceptibility to EMI depending upon the electrical shielding properties of controllers against various intensities and frequencies of EMI. We use a Markov-chain model to describe burst upsets when EMI is present. We then modify a system dynamic equation by including the stochastic features (occurrences/magnitudes) of these upsets, and examine the condition of system stability for the mean behaviors of the modified equaKang G. Shin Real-Time Computing Laboratory Dept. of Elec. Eng. and Comput. Sci. The University of Michigan Ann Arbor, MI 48109-21226 Tel:313-763-0391 Email:kgshin@eecs.umich.edu

tion. The derived information about the required level of the stationary probability of upsets is a key to the design and verification of the integrity of reliable controllers. We also present a simple experiment emulating EMI on data transmission to estimate the necessary parameters and a demonstration example about system stability.

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#### 1. INTRODUCTION

An environmental disruption like Electro-Magnetic Interference (EMI) is a main source of external faults on a digital controller-computer, which resides in the

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feedback loop to periodically run programmed tasks using sensory data from the controlled process and to provide actuators with the control inputs at regular time frames (defined by  $T_s$ ).

Unlike most internal faults being permanent or intermittent due to physical defects (e.g., broken, short, or loose connections), external faults are likely to be transient and cause primarily functional error modes in a digital system. In other words, these upset phenomena result primarily in (i) a change in data values of the input/output circuitry (including the interfaces such as A/D and D/A between actuators/sensors and computing units), (ii) logic changes on the data bus, address bus, and control lines of the microprocessors, (iii) logic changes in registers of the CPU, and (iv) logic changes in the ALU inside the CPU. The outcomes of these abnormalities (called upsets)<sup>1</sup> induce such computer failures as not updating any control input or generating erroneous control inputs during one or more sampling periods. The stationary occurrences of these upsets may lead to the loss of system stability either if their active duration exceeds a certain limit [2] or if they occur too frequently.

In [5], EMI effects on modern digital systems were shown substantial according to a questionnaire distributed to exports to gather the data. Some information on the frequency of EMI occurrences was presented via a subjective study focusing on externally-generated EMI from such man-made emitters of EM energy as radars and broadcasting transmitters. In [1], a methodology for performing an upset (failure) test in a laboratory environment on a multi-channel control system was presented with a case study of a fault-tolerant electronic engine control system. The test primarily assessed the effects of electromagnetic disturbances but lacked analytic tools either predicting or interpreting the results. In [2, 4], we previously derived the maximum allowable duration of computer failures (called the control system deadline) for linear time-invariant control systems. Although the work analyzed the effects of the duration of computer failures on system stability, it can not capture the effects of stationary occurrences of environmental interferences on the control system (computer failure and thus system stability) - the computer sometimes operates normally in spite of the presence of EMI. It is difficult to validate the theoretic results in the paper with experiments.

In this paper, we investigate the effects of EMI both on the controller computer and on the control plant at the same time by using a Markov-chain model, and derive the stationary probability of controller upsets, which represents not only the level of suscep*tibility* of the system against EMI but also the occurrence rate of computer failures due to EMI. We also mention and formalize the aspects/features of upsets by using certain probability-distribution models for control input disturbances. We modify the system dynamic equation accounting for controller upsets and input disturbances, and examine stability-in-the-mean in the presence of stationary-occurring EMI.

This information is invaluable to the design and verification of the integrity of reliable controllers, because the probability of controller upsets can capture the relation between the occurrence/recovery of computer failures and the level of faultavoidance/fault-tolerance of the computers. Note that the occurrence of computer failure(s) induced by EMI depends on the electrical shielding property and the structural material against various intensities and fre-

<sup>&</sup>lt;sup>1</sup>Throughout this paper, we use the terms *upsets*, *controller perturbations*, and *computer failures* to mean the same.

quencies of EMI, which prevent an exterior electromagnetic field from penetrating into computer modules, as well as on the adopted error-/failure- handling method to recover from a temporary failure.<sup>2</sup>

Our results can also be validated via a experiment coupling electromagnetic fields directly into computer systems and counting number of time frames  $(T_s)$  producing incorrect outputs. We specifically derive the maximum value of the stationary probability of controller abnormalities (faulty states) that can maintain system stability under given characteristics of EMI (intensity, occurrence, and duration), and present a simple example disturbing data-transmission lines through sinusoidal EMI with various frequencies and voltages in a reverberation chamber so as to measure the parameters required for obtaining the probability of controller perturbations.

Section 2 presents problem statements describing generic properties of EMI, necessary assumptions, and a model of EMI behaviors. Section 3 computes the probability of controller upsets and formalizes the control input disturbances. We derive the condition not to lose system stability using those parameters. In Section 4, we demonstrate our analytic work using a simple experiment which should be refined in future. The paper concludes with Section 5.

### 2. PROBLEM STATEMENT

Digital controller computers have made significant advances in automating the command, control and communication functions

in a real-time control system like aircraft. In such an adverse operating environment as High Intensity Radiated Field (HIRF), EM field may cause an analog electrical signal/noise to be induced and propagated to on-board electronic equipments. Unlike an analog system having only a tiny effect of such an small pulse of noise mixed or added due to its larger signal magnitude, a large error may occur in case of a digital signal if the noise pulse flips the (most) significant bit. In other words, some digital microelectronic devices are more susceptible to unwanted noise than older analog electronics due to shrinking device size, lower switching energy, and higher-speed operations. Thus, evaluating susceptibility of the digital controller computers to EMI is a key to the development and verification of critical control systems.

Common phenomena caused by EMIinduced signals are changes of data values in the I/O circuitry including A/D and D/A converters and logic changes on the data/address bus, control lines, registers, or ALUs within the CPU. These effects primarily result in temporal functional error modes leading to control-law computation errors on the closed loop [1, 3]. Since a controller computer generally executes predefined control jobs with sensed inputs and provides actuators with the control inputs repeatedly during periodic time frames  $(T_s)$ , such temporal upsets result in generating erroneous control inputs or input disturbances due to wrong computations during a certain number of time frames. (Note that in case the computer not producing any output within  $T_s$ , either an incorrect value randomly-produced or the previous one is taken as a control input by the actuator.) It is also noteworthy that the computer sometimes operates normally in spite of the presence of EMI.

In this paper, we assume that the only effect

 $<sup>^{2}</sup>$ In this paper, we focus on externally-induced faults due to EMI, which affect transiently without any component damage and disappear in a certain time.

of EMI on the controller is a functional error mode (upset, i.e., malfunction of control-law calculation) without any permanent damage, which comes back to the normal operation in a certain time without an additional recovery action.<sup>3</sup>

In addition, we assume that no upset occurs in the absence of EMI — although some internal faults due to manufacturing defects or wearing effects can still occur/persist, those are not our concern in analyzing the effects of EMI. On the other hand, EMI is generally characterized by a long latent period followed by a relatively short period of presence. We assume that EMI arrivals follow a time-invariant Poisson process with a rate  $\lambda_e$  and each arrival remains active for an exponentially-distributed random period with a rate  $\mu_e$ . Consequently, environmental disruptions arrive at an exponential rate  $\lambda_e$ and disappear after an active duration with mean  $\frac{1}{\mu_e}$ .

#### 3. Effects of EMI on Stability

As mentioned earlier, EMI generally induces controller perturbations, which may be repeated during a certain number of time frames. It is also true that the controller may sometimes perform correctly even in the presence of EMI. The control plant can survive those repeated controller-upsets due to its plant dynamics/inertia [2]. However, stationary occurrences of these upsets may lead to the loss of system stability, either if their active duration exceeds a certain limit [4] or if they do too often occur. In this paper, we evaluate the susceptibility of the controller against EMI by computing the stationary probability of controller upsets (computer failures) due to EMI, from which one can obtain information about the occurrence rate of EMI upsets in controller computers. We then derive the maximum probability of upsets due to EMI maintaining system stability.

# Stationary Probability of Controller Upsets Due to EMI:

We begin with a handy model assuming that the probability of upset(s) during one time frame  $T_s$  in the presence of EMI is a constant, p, independent of whether the previous state is faulty or not — a more realistic model will be later investigated covering burst upsets in the presence of EMI. A Markov chain with two states is used to describe these aspects. The two states will be called A (for EMI Absence) and E (for EMI*Existence*). In state A no upset occurs, while in state E an upset occurs with probability p like the feature of tossing a biased coin. After producing an event (upset or no upset), the Markov chain makes a transition to prepare next event. The transition probabilities  $q = \Pr(A \rightarrow E)$  and  $r = \Pr(E \rightarrow A)$ will be derived by the probability model of EMI behaviors assumed in Section 2. That is,

$$q = \int_{0}^{T_{s}} \lambda_{e} e^{-\lambda_{e} t} dt = 1 - e^{-\lambda_{e} T_{s}},$$
  
$$r = \int_{0}^{T_{s}} \mu_{e} e^{-\mu_{e} t} dt = 1 - e^{-\mu_{e} T_{s}}.$$
 (3.1)

Fig. 1 is a transition diagram of the Markov chain, where runs of A will alternate with runs of E and the run lengths in a row have geometric distributions with mean 1/q for the A-runs and mean 1/r for the E-runs. (Note that although one might construct more accurate models, which is so complicated and may be useless without appropriate (vast) statistical data, we justify the

<sup>&</sup>lt;sup>3</sup>Unless we use this assumption, we should consider proper fault-tolerance methods for detecting faults/failures and handling safe recoveries from permanent ones as well as quick recoveries even from transient ones.



Figure 1. A Simple Markov-Chain Model Having Two States According to EMI Existence

geometric distributions of those runs under the assumption of independent events among different time-frames for mathematical simplicity.) Let  $\Pi_f$  be the stationary probability of upset occurrences. Since the stationary probability of state E, i.e., the fraction of time spent in E, is simply obtained by  $\Pr(E) = q/(q+r)$  and an upset occurs only in state E with probability p,  $\Pi_f$  is equal to:

$$\Pi_f = p \Pr(E) = p \frac{q}{q+r}.$$
 (3.2)

Now, we consider a more realistic model generating burst upsets in the presence of EMI, for which the state E should be classified into N (for No upset/failure) and F (for Upset) according to whether or not an upset occurs. Before dealing with this model, we need to consider the event of EMI occurrences in detail, as depicted in Fig. 2-(a). If EMI occurs, the state moves from A into E. However, the state E in the case is a dummy state, whose holding time is zero, i.e., the state goes to either N or F instantaneously passing through E. Thus, the state E is no longer necessary, and the diagram of Fig. 2-(b) can capture all these phenomena. This Markov chain with three states can explain the burst upsets in the presence of EMI. In other words, the state F must tend to persist to simulate burst upsets with large  $p_2$ , and  $p_2 > p_1$  since the upset during next time frame is more likely to occur in a currently-upset state than no upset state. From this model, we can also derive the stationary probability of the state producing upsets.

Let  $\Pi_A$ ,  $\Pi_N$ , and  $\Pi_F$  be the stationary probabilities (i.e., the fractions of time spent) of the states A, N, and F, respectively. Then, those are obtained by solving:

$$\Pi_{A} = (1-q)\Pi_{A} + r\Pi_{N} + r\Pi_{F},$$

$$\Pi_{N} = q(1-p_{1})\Pi_{A} + (1-r)(1-p_{1})\Pi_{N}$$

$$+ (1-r)(1-p_{2})\Pi_{F},$$

$$\Pi_{F} = qp_{1}\Pi_{A} + p_{1}(1-r)\Pi_{N} + p_{2}(1-r)\Pi_{F},$$

$$(3.3)$$

where  $\Pi_A + \Pi_N + \Pi_F = 1$ . In reality, the stationary probability of EMI presence,  $\Pr(E)$ , is equal to  $\Pi_N + \Pi_F$ , i.e.:

$$\Pi_N + \Pi_F = \frac{q}{q+r} \quad \text{and} \quad \Pi_A = \frac{r}{q+r}. \quad (3.4)$$

Thus, the probability of state F can be computed by plugging Eq. (3.4) into Eq. (3.3):

$$\Pi_{F} = q p_{1} \frac{r}{q+r} + p_{1}(1-r) \left(\frac{q}{q+r} - \Pi_{F}\right) + p_{2}(1-r) \Pi_{F}, = (1-r)(p_{2}-p_{1}) \Pi_{F} + \frac{q p_{1}}{q+r} \left[r+(1-r)\right] = \frac{q}{(q+r)} \frac{p_{1}}{[1-(1-r)(p_{2}-p_{1})]}.$$
(3.5)



Figure 2. Markov-Chains (a): with an Instant State E and (b): with Three States.

Consequently, we obtain the stationary probability of upset occurrences due to EMI by Eq. (3.5), where q and r (equivalently,  $\lambda_e$  and  $\mu_e$ ) can be estimated using field data about EMI behaviors [5].

The parameters,  $p_1$  and  $p_2$ , can also be estimated experimentally. Although  $p_1$  and  $p_2$ are not directly observable, these parameters can be deduced from statistical measurements using other easily-estimated parameters. We consider an experiment emulating EMI and coupling electromagnetic fields directly into computer systems, where we observe the time frame having an upset in the presence of EMI and count the number of frames (upsets) producing incorrect outputs. For the analysis, we use the diagram of Fig. 2-(b) with q = 1 and r = 0, which implies that EMI always exists. The samples to be directly measured are the numbers of consecutive frames during which no upset occurs (runs of N in a row) and the numbers of consecutive frames during which

upsets persist (runs of F in a row). For example, if we obtain the sets of the samples for both cases, respectively, like:

$$\{a_1, a_2, \cdots, a_{j_1}\}$$
 and  $\{b_1, b_2, \cdots, b_{j_2}\},$ 

where  $j_1$  and  $j_2$  are the total numbers of samples for both cases, respectively. Let  $s_1 = \sum_{i=1}^{j_1} a_i$  and  $s_2 = \sum_{i=1}^{j_2} b_i$  be the sums of those samples, respectively, then the total number of frames is equal to  $s_1 + s_2$ . Let  $\bar{n_1}$ and  $\bar{n_2}$  be defined as the mean values of the samples for both run lengths collected from the experiment, then

$$\bar{n_1} = \frac{s_1}{j_1}$$
 and  $\bar{n_2} = \frac{s_2}{j_2}$ . (3.6)

We can also derive these parameters analytically from the diagram of Fig. 2-(b) with q = 1 and r = 0. The mean length of *F*runs is:

$$E(n_2) = \sum_{k=1}^{\infty} k p_2^{k-1} (1-p_2) = (1-p_2) \sum_{k=1}^{\infty} k p_2^{k-1}$$
$$= (1-p_2) \sum_{k=1}^{\infty} (p_2^k)' = (1-p_2) \left( \sum_{k=1}^{\infty} p_2^k \right)'$$

$$= (1-p_2) \left(-1+\sum_{k=0}^{\infty} p_2^k\right)'$$
  
=  $(1-p_2) \left(-1+\frac{1}{1-p_2}\right) = \frac{1}{1-p_2}.$   
(3.7)

Solving Eqs. (3.6) and (3.7), we can estimate the desired parameters for  $p_1$  and  $p_2$  in terms of  $\bar{n_1}$  and  $\bar{n_2}$ , whose estimators are defined by  $\bar{p_1}$  and  $\bar{p_2}$ , respectively, then:

$$\bar{n_2} = \frac{1}{1 - \bar{p_2}} \implies \bar{p_2} = 1 - \frac{1}{\bar{n_2}} = 1 - \frac{j_2}{s_2}, \bar{n_1} = \frac{1}{\bar{p_1}} \implies \bar{p_1} = \frac{1}{\bar{n_1}} = \frac{j_1}{s_1},$$
(3.8)

Features and Effects of Upsets on System Stability:

In this paper, we consider a system described by a linear discrete time-invariant recurrence equation as given by:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k); \ \mathbf{x}(k_0) = \mathbf{x}_0,$$
(3.9)

where k indicates each time frame (e.g., one sampling interval  $T_s$ ) from an initial time  $k_0$ . Here  $\mathbf{x}(k) \in \mathcal{R}^n$  is the state of the system,  $\mathbf{u}(k) \in \mathcal{R}^m$  the applied control, and the matrices **A** and **B** are bounded, which can be obtained from the corresponding continuoustime model [4].

We start our analysis by formally defining system stability in terms of the concept of exponential convergence of a sequence  $\mathbf{x}[k_0, \cdots]$ . That is,  $\mathbf{x}(\cdot)$  converges exponentially to zero if there exist positive constants  $\alpha$  and M such that  $||\mathbf{x}(k)|| < Me^{-\alpha(k-k_0)}$  for all  $k > k_0$ . The system is defined as stablein-the-mean if  $E[\mathbf{x}(k)]$ , which is the expected value of the state at k-th time frame, converges exponentially to a certain equilibrium state of the system. Since some recent and future systems such as highly fuel-efficient "fly-by-wire" aircraft — which are likely to be intrinsically unstable to achieve other purposes or go to the edge of stability — demand the reliable control actions to maintain stability (*stabilizable*), the frequent upsets of the controller induced due to EMI may affect seriously system stability. We specifically attend to a system using a certain feedback control input,  $\mathbf{u}(k) = \mathbf{Fx}(k)$ , for stabilizing the system matrix **A** (and optimizing a certain performance index).

As described earlier, during an upset the controller computer fails in providing the physical actuator with correct control inputs due to either (i) control-law calculation errors resulting from logic changes inside processors or (ii) transmission disturbances caused by data changes on sensors/actuators lines or on I/O circuitry containing the interfaces such as A/D and D/A converters. In other words, the control input during an upset becomes  $\mathbf{u} + \boldsymbol{\Delta}$  (additive disturbance) or  $(\mathbf{I} + \boldsymbol{\Delta})\mathbf{u}$  (multiplicative disturbance), where I is a identity matrix and  $\Delta$ is a diagonal matrix with random-sequence elements,  $diag[\mathbf{\Delta}]_i = \Delta_i$ , modeled by the bounded outputs of certain dynamic systems with white-noise sequences. Let  $\mathbf{u}_a(k)$  be an actual control input, which becomes an desirable one  $\mathbf{Fx}(k)$  or a disturbed one according to whether or not an upset occurs. The disturbed one can also be described in detail as follows:

- in case of control-law calculation errors,  $\mathbf{u}_a = (\mathbf{F} + \Delta_{\mathbf{f}})\mathbf{x}$  or  $(\mathbf{I} + \Delta_{\mathbf{f}})\mathbf{F}\mathbf{x}$ ,
- in case of transmission errors on the sensor line,  $\mathbf{u}_{\alpha} = \mathbf{F}(\mathbf{x} + \boldsymbol{\Delta}_{s})$  or  $\mathbf{F}(\mathbf{I} + \boldsymbol{\Delta}_{s})\mathbf{x}$ ,
- in case of transmission errors on the actuator line,  $\mathbf{u}_a = \mathbf{F}\mathbf{x} + \boldsymbol{\Delta}_{\mathbf{a}}$  or  $(\mathbf{I} + \boldsymbol{\Delta}_{\mathbf{a}})\mathbf{F}\mathbf{x}$ ,

where  $\Delta_s$ ,  $\Delta_f$ , and  $\Delta_a$  are the same kinds of matrix as  $\Delta$  being independent of one another, and the mean of each random sequence is given, measurable through some experiments, as  $\bar{\Delta}_s$ ,  $\bar{\Delta}_f$ , and  $\bar{\Delta}_a$ , respectively. Here, we are treating only the multiplicative disturbances because the theory of additive disturbances is also well understood. To modify the system dynamic equation (Eq. (3.9)) accounting for the upset effects, we represent  $\mathbf{u}_a(k)$  affected by an upset as:

$$\mathbf{u}_{a}(k) = (\mathbf{I} + \boldsymbol{\Delta}_{\mathbf{a}})(\mathbf{I} + \boldsymbol{\Delta}_{\mathbf{f}})\mathbf{F}(\mathbf{I} + \boldsymbol{\Delta}_{\mathbf{s}})\mathbf{x}(k),$$
(3.10)

which covers all three features described above.<sup>4</sup> If the system has an ideal scheme detecting failures perfectly and instantaneously, one can suggest better control strategies to hold the control inputs by the previous values or to set  $\mathbf{u}_{\alpha}(k) = \mathbf{0}$ (i.e.,  $\boldsymbol{\Delta} = -\mathbf{I}$  for the multiplicative disturbances) during the computer failures. However, we consider general and practical cases of random (arbitrary) control inputs generated due to the computer failures.

Let  $\pi$  indicate an upset indicator, which will be 0 or 1 according to the occurrence of upset at each time frame. By using this sequence of binary upset digits, we can rewrite the system dynamic equation as follows:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}_{a}(k) \\ &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}(1-\pi)\mathbf{F}\mathbf{x}(k) + \\ \pi \mathbf{B}(\mathbf{I} + \boldsymbol{\Delta}_{\mathbf{a}})(\mathbf{I} + \boldsymbol{\Delta}_{\mathbf{f}})\mathbf{F}(\mathbf{I} + \boldsymbol{\Delta}_{\mathbf{s}})\mathbf{x}(k) \\ &= [\mathbf{A} + \mathbf{B}(1-\pi)\mathbf{F} + \\ \pi \mathbf{B}(\mathbf{I} + \boldsymbol{\Delta}_{\mathbf{a}})(\mathbf{I} + \boldsymbol{\Delta}_{\mathbf{f}})\mathbf{F}(\mathbf{I} + \boldsymbol{\Delta}_{\mathbf{s}})]\mathbf{x}(k). \end{aligned}$$

$$(3.11)$$

We compute the mean of this stochastic equation using the results in the previous section, i.e.,  $E(\pi) = \Pi_F$ . Then, the mean of the state vector  $\bar{\mathbf{x}}$  evolves according to:

$$\bar{\mathbf{x}}(k+1) = [\mathbf{A} + \mathbf{B}(1 - \Pi_F)\mathbf{F} +$$

$$\Pi_{F} \mathbf{B}(\mathbf{I} + \bar{\boldsymbol{\Delta}}_{\mathbf{a}})(\mathbf{I} + \bar{\boldsymbol{\Delta}}_{\mathbf{f}})\mathbf{F}(\mathbf{I} + \bar{\boldsymbol{\Delta}}_{\mathbf{s}})]\mathbf{x}(k) = \left[\mathbf{A} + \mathbf{B}\left(1 - \frac{qp_{1}}{(q+r)[1 - (1-r)(p_{2} - p_{1})]}\right)\mathbf{F} + \frac{qp_{1}}{(q+r)[1 - (1-r)(p_{2} - p_{1})]}\mathbf{B}(\mathbf{I} + \bar{\boldsymbol{\Delta}}_{\mathbf{a}})\right] \mathbf{F} + \frac{qp_{1}}{(\mathbf{I} + \bar{\boldsymbol{\Delta}}_{\mathbf{f}})\mathbf{F}(\mathbf{I} + \bar{\boldsymbol{\Delta}}_{\mathbf{s}})} \mathbf{F}(\mathbf{I} + \bar{\boldsymbol{\Delta}}_{\mathbf{s}}) \mathbf{F$$

We can derive the maximum value of  $\Pi_F$ maintaining system stability by examining the eigenvalues of Eq. (3.12), which clearly depend on the probability models of disturbances  $\{\Delta_f, \Delta_s, \Delta_a\}$ . We guess that as the probability of upset occurrences increases the stability property is less preserved by comparing the maximum values of eigenvalues for various values of  $\Pi_F$ . The condition of  $\Pi_F$  for asymptotic system stability, which is a stronger condition in the engineering sense, can also be obtained by using a theory in [6]. Since the systems are subjected to a countably infinite number of system matrices, supposed to be independent identically distributed, according to Eq. (3.11), asymptotic stability can be justified by examining the mean value of the norms of the system matrices. In terms of the theory developed in [6], we can easily derive the maximum value of  $\Pi_F$  maintaining the mean of the norms less than one and thus retaining system stability with probability one. From the requirements of  $\Pi_F$ , we can consequently obtain the conditions of  $p_1$  and  $p_2$ , which are determined by the electrical shielding properties against various intensities and frequencies of EMI.

## 4. Example

Due to the problem of feasibility in experimental facilities, we conduct a primitive experiment generating only some transmission disturbances. More practical and elaborate experiments causing control-law calculation

<sup>&</sup>lt;sup>4</sup>We propose this form in the sense of multiplicative disturbances because of its simplicity for our analysis.

errors by reverberating EMI on the computing processors are currently being planned. In the experiment, the electrical isolation of experimental facilities, e.g., detecting upsets (transmission errors), is achieved using fiber optics, which is described in detail with a full explanation of data analysis [7].

First, we obtain samples for the run lengths for both states, N and F, via the experiment emulating EMI and coupling EMI directly on the transmission lines in a HIRF test chamber. We sent both deterministic (sawtooth and ramp) signals, and random signals generated according to uniform and normal distributions, where one signal set consists of 8192 bytes, through the lines in various EM fields, and compared those input signals with the received signals to determine the state at each time frame. Using the mean of the samples of run lengths in a row, we estimate  $p_1$  and  $p_2$  based on Eq. (3.8), which in turn give the estimated value of the probability of upsets according to Eq. (3.5).

**Table 1.**  $(\bar{n_0}, \bar{n_1})$  and  $(\bar{p_1}, \bar{p_2})$ 

Freq:Power	$(ar{n_0},ar{n_1})$	$( ilde{p_1},  ilde{p_2})$
525:2	(12, 7375)	(0.000136, 0.917)
525:5	(18, 6966)	(0.000144, 0.945)
525:10	(16, 5340)	(0.000187,0.938)
525:50	(14, 16)	(0.000608, 0.928)
550:2	(13, 6149)	(0.000163, 0.925)
550:5	(10, 6972)	(0.000143, 0.904)
550:10	(7, 5988)	(0.000167,0.861)
550:50	(25, 1232)	(0.000812, 0.960)

In Table 1, we present the estimated  $p_1$  and  $p_2$  in the presence of EMI having two frequencies such as 525 and 550 [MHz] for various power levels [W], to which transmission upsets are most sensitive. Other frequencies (such as 475, 500, and 575 [MHz]) did not

induce any upset regardless of the power levels. We guess that there are critical frequencies of EMI contributing more significantly to upsets, depending upon the frequency of the carrier signal and the type/geometry of placement of the transmission lines in the chamber. It is certain that even low amplitude signals at frequencies near the clock speeds, or the information bandwidth, of digital circuitry may seriously affect the system. The results showing  $(1-p_1) \gg p_1$ and  $p_2 \gg (1-p_2)$  support our model covering burst upsets. Suppose that the parameters of EMI behaviors are given by  $\lambda_e T_s =$  $9.26 \times 10^5$  and  $\mu_e T_s = 0.033$ . For example, if  $T_s$  is one second, the mean rate of EMI occurrences is 3 hours and the mean duration is 30 seconds. We then estimate  $\Pi_F$  for various frequencies [MHz] and power levels [W], as given in Table 2.

Table 2. Upset Probability  $\Pi_F$ 

Freq:Power	$\Pi_F$
525:2	$3.367 imes10^{-6}$
525:5	$4.691 \times 10^{-6}$
525:10	$5.678  imes 10^{-6}$
525:50	$3.219 imes10^{-6}$
550:2	$1.062  imes 10^{-3}$
550:5	$4.338 imes10^{-6}$
550:10	$2.807 imes10^{-6}$
550:50	$3.160 imes10^{-5}$

We now present a simple example to determine how often an controller upset can occur in the system to maintain system stabilityin-the-mean in terms of Eq. (3.12). The dynamic behavior of the altitude of a spinning satellite is described in terms of the longterm control of the roll ( $\varphi$ ) and yaw ( $\psi$ ) angles, which is based on the dynamic coupling resulting from the rotation of the satellite around the earth:

$$\dot{\varphi}_x = 2.1\varphi_x + 1.5\psi_z + 0.1u_x + 0.2u_z,$$

$$\dot{\psi}_z = -2.3\psi_z + 0.2u_x + 0.1u_z,$$
 (4.13)

where the coefficients depend upon the orbital frequency, i.e., the angular velocity of the satellite with respect to the inertial frame, and  $u_x$  and  $u_z$  are control signals. The goal of the control is to maintain a desired orientation of the satellite in the orbit around the earth (the stabilization problem) with the minimum-control effort, which results in the optimal (feedback) control gain matrix **F** by minimizing a quadratic performance index:

$$J = \frac{1}{2} \left( \sum_{k=0}^{k_f - 1} \left[ \mathbf{x}^T(k) \mathbf{Q}(k) + \mathbf{u}^T(k) \mathbf{R}(k) \right] + \mathbf{x}^T(k_f) \mathbf{Q}(k_f) \right).$$
(4.14)

Suppose that  $\mathbf{Q} = 2\mathbf{I}$  and  $\mathbf{R} = 5\mathbf{I}$  are determined by the control objective of interest and  $T_s = 1$ . The corresponding coefficient matrices are then:

$$\mathbf{A} = \begin{bmatrix} 8.1660 & 2.7490 \\ 0 & 0.1003 \end{bmatrix};$$

$$\mathbf{B} = \begin{bmatrix} 0.5470 & 0.7855 \\ 0.0782 & 0.0391 \end{bmatrix};$$

$$\mathbf{F} = \begin{bmatrix} 4.7623 & 1.6249 \\ 6.6508 & 2.2661 \end{bmatrix}.$$

This feedback control changes the eigenvalues from  $\{8.166, 0.1003\}$  to  $\{0.1102 \pm 0.0045j\}$ , thus stabilizing the satellite. When we consider some EMI effects generating upsets in the controller, the eigenvalues are changed by varying the frequency in stationary occurrences of upsets. In Table 3, the maximum values of the eigenvalues are shown while varying  $\Pi_F$ , given that  $\Delta_f = 0$ ,  $\bar{\Delta}_a = -10\mathbf{I}$ , and  $\bar{\Delta}_s = 10\mathbf{I}$  considering only transmission disturbances. We see that the stability property is less preserved as  $\Pi_F$  increases. When  $\Pi_f \geq 0.001092$ , the system has at least one eigenvalue greater than or

Table 3. Maximum Eigenvalues

$\Pi_F$	Max. of Eigenvalues
$1 \times 10^{6}$	0.1319
$5 \times 10^{6}$	0.1344
$1 imes 10^5$	0.1376
$5 \times 10^5$	0.1662
$1 \times 10^4$	0.2046
$5 \times 10^{4}$	0.5242
$1 \times 10^3$	0.9260
$2 \times 10^3$	1.7303

equal to 1, implying the loss of system stability. As a result, one should design the controller tolerable sufficiently against EMI to have the corresponding  $p_1$  and  $p_2$ , or equivalently to meet the condition  $\Pi_f < 0.001092$ retaining stability.

#### 5. CONCLUSION

In this paper, we investigated the effects of EMI both on the controller computer and on the control plant simultaneously. First we derived the stationary probability of upsets induced due to EMI, which represents the level of susceptibility of the controller to EMI depending upon the EMI behaviors and the shielding properties of the materials and structures of the controller against EMI. For a realistic model covering burst upsets, we classified the state of EMI presence into two states according to whether an upset occurs or not. We also examined system stability by using a stochastic dynamic equation modified to account for the effects of upsets. The results showed the effects on EMI on the control plant indirectly through the probability of upsets in the controller. We presented an example examining system stability and a handy experiment estimating the necessary parameters to demonstrate our theoretical work. This kind of experiment should be more realistic by reverberating EMI on a real controller-computer and simulating an actual avionic system, though.

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